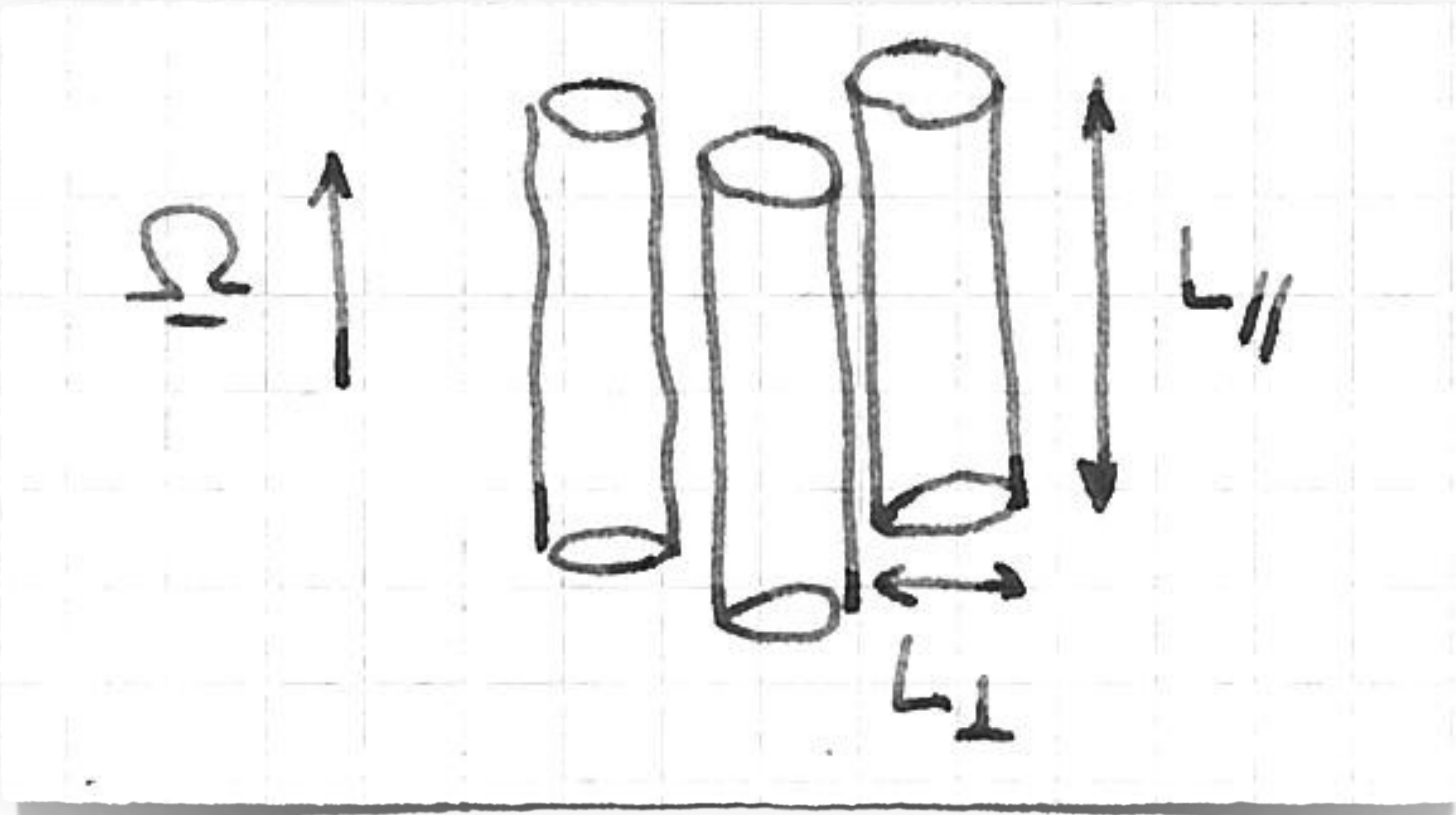
Rotating turbulence

A nice review: Godeferd & Moisy (2015)



p/ρ_0

$$\frac{\partial \underline{v}}{\partial t} + \underline{v}\cdot\nabla\underline{v} = -2\underline{\Omega}\times\underline{v} - \underline{\nabla} p + \nu\nabla^2\underline{v}$$

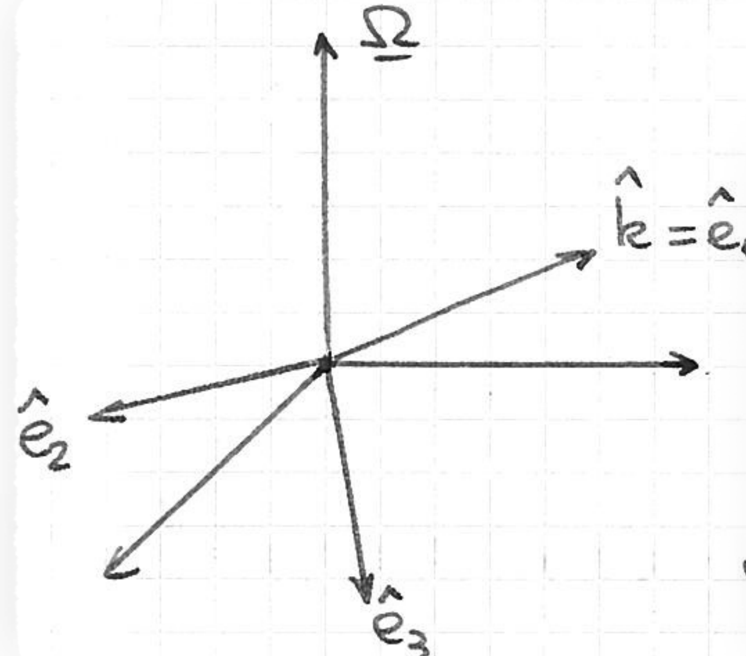
$$\begin{cases} \underline{v}(\underline{x},t) = \sum_{\underline{k}} \hat{\underline{v}}_{\underline{k}}(t)\, e^{i\underline{k}\cdot\underline{x}} \\ p(\underline{x},t) = \sum_{\underline{k}} \hat{p}_{\underline{k}}(t)\, e^{i\underline{k}\cdot\underline{x}} \end{cases}$$

$$\Rightarrow \quad \partial_t \hat{\underline{v}}_k = -\sum_{\underline{p}+\underline{q}=k} (\hat{\underline{v}}_p\cdot i\underline{q})\,\hat{\underline{v}}_q - 2\Omega\times\hat{\underline{v}}_k - i\underline{k}\hat{p}_k - \nu k^2\hat{\underline{v}}_k \qquad (1)$$

$$\underline{v} = \hat{\underline{v}}_{\underline{k}}\, e^{i(\underline{k}\cdot\underline{x} - \sigma_s t)}$$

$$\sigma_s = 2s\,\frac{\underline{\Omega}\cdot\underline{k}}{k} = 2s\,\underline{\Omega}\cdot\hat{\underline{k}}$$

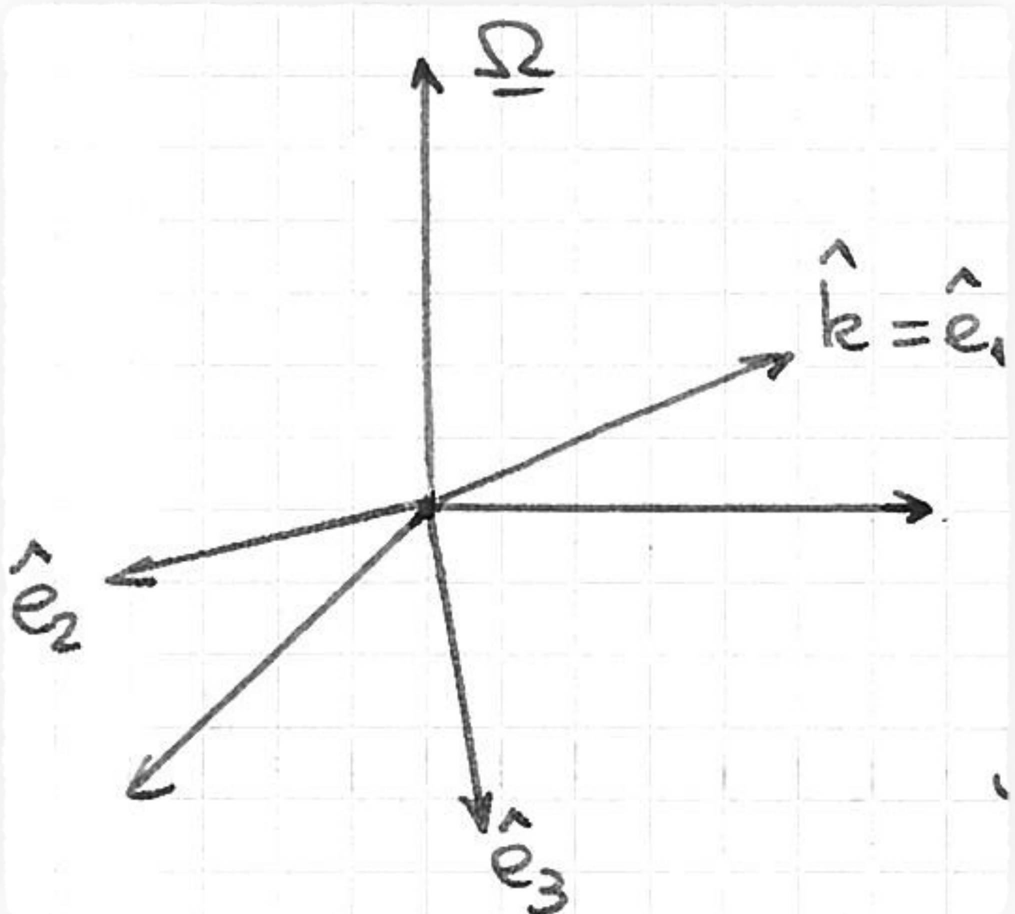
$$i\underline{k}\times\hat{\underline{v}}_k = \mp\underline{v}_0 k = -sk\underline{v}_0$$



$$\hat{e}_1 = \hat{\underline{k}} = \frac{\underline{k}}{|\underline{k}|}$$

$$\hat{e}_2 = \frac{\underline{k}\times\underline{\Omega}}{|\underline{k}\times\underline{\Omega}|}$$

$$\hat{e}_3 = \hat{e}_1\times\hat{e}_2$$



$$\hat{\underline{u}}_{\underline{k},s} = b_s(\underline{k})\,\underline{h}_s(\underline{k})$$

$$\boxed{\underline{h}_s(k) = \hat{e}_3 + is\hat{e}_2}$$

$$\begin{aligned} i\underline{k} \times (\hat{e}_3 + is\hat{e}_2) &= ik(\hat{e}_1 \times \hat{e}_3 + is\,\hat{e}_1 \times \hat{e}_2) = \\ &= ik(-\hat{e}_2 + is\hat{e}_3) = -k(s\hat{e}_3 + i\hat{e}_2) \\ &= -sk(\hat{e}_3 + is\hat{e}_2) \quad . \end{aligned}$$

$$\underline{u} = \sum_{\underline{k}} \sum_{s_k = \pm 1} b_{s_k}(\underline{k})\,\underline{h}_{s_k}(\underline{k})\, e^{i(\underline{k}\cdot\underline{x} - \sigma_{s_k} t)}$$

$$R_o = \frac{U}{2L\Omega} = \frac{\tau_\Omega}{\tau_0} \ll 1 \quad \Rightarrow \quad \tau_\Omega \ll \tau_0$$

$$\tau_0 = L/U$$

Slow modulation

$$\underline{v} = \sum_{\underline{k}, s_k} b_{s_k}(\underline{k}, t)\, \underline{h}_{s_k}(\underline{k})\, e^{i(\underline{k}\cdot\underline{x} - \sigma_{s_k} t)} =$$

$$= \sum a_{s_k}(\underline{k}, t)\, \underline{h}_{s_k}(\underline{k})\, e^{i\underline{k}\cdot\underline{x}}$$

$$a_{s_k}(\underline{k}, t) = b_{s_k}(\underline{k}, t)\, e^{-i\sigma_{s_k} t}$$

$$\underline{h}_{s_k}(\underline{k})\, \partial_t a_{s_k}(\underline{k}) = -\sum_{\substack{\underline{k}=\underline{p}+\underline{q} \\ s_p, s_q}} \left(\underline{h}_{s_p}(\underline{p})\cdot i\underline{q}\right) \underline{h}_{s_q}(\underline{q})\, a_{s_p}(\underline{p})\, a_{s_q}(\underline{q}) -$$

$$- 2\underline{\Omega} \times \underline{h}_{s_k}(\underline{k})\, a_{s_k}(\underline{k}) - \nu k^2\, \underline{h}_{s_k}(\underline{k})\, a_{s_k}(\underline{k}) - i\,\underline{k}\,\hat{p}_{\underline{k}}$$

$$\Rightarrow 2\partial_t a_{s_k}(\underline{k}) = -\sum_{\substack{\underline{k}=\underline{p}+\underline{q}\\ s_p, s_q}} \underline{h}^*_{s_k}(\underline{k})\cdot\left\{\left[\underline{h}_{s_p}(\underline{p})\cdot i\underline{q}\right]\underline{h}_{s_q}\right\} a_{s_p}(\underline{p})\, a_{s_q}(\underline{q})$$

$$- 2\nu k^2 a_{s_k}(\underline{k}) - 2 i \sigma_{s_k} a_{s_k}(\underline{k})$$

$$\Rightarrow \left(\partial_t + \nu k^2 + i\sigma_{s_k}\right) a_{s_k}(\underline{k}) = +\frac{1}{2}\sum_{\substack{\underline{k}+\underline{p}+\underline{q}=0\\ s_p, s_q}} \underbrace{\underline{h}^*_{s_k}(\underline{k})\cdot\left\{\left[\underline{h}^*_{s_p}(\underline{p})\cdot i\underline{q}\right]\underline{h}^*_{s_q}(\underline{q})\right\}}_{\parallel \atop C_{kpq}} a^*_{s_p}(\underline{p})\, a^*_{s_q}(\underline{q}) \qquad (2)$$

$$\boxed{C_{kpq} = \frac{1}{2}\left(s_q q - s_p p\right)\, \underline{h}^*_{s_k}(\underline{k})\cdot\left[\underline{h}^*_{s_p}(\underline{p})\times \underline{h}^*_{s_q}(\underline{q})\right] = \left(s_q q - s_p p\right) g}$$

$$a_{s_k}(\underline{k}, t) = b_{s_k}(\underline{k}, t)\, e^{-i\sigma_{s_k} t}$$

$$\partial_t a_{s_k} = -i\sigma_{s_k} a_{s_k} + e^{-i\sigma_{s_k} t}\partial_t b_{s_k}$$

$$\boxed{\left(\frac{\partial}{\partial t} + \nu k^2\right) b_{s_k}(\underline{k}) = \frac{1}{2}\sum_{\substack{\underline{k}+\underline{p}+\underline{q}=0\\ s_p, s_q}} C_{kpq}\, b^*_{s_p}(\underline{p})\, b^*_{s_q}(q)\, e^{i\left(\sigma_{s_k}+\sigma_{s_p}+\sigma_{s_q}\right)t}}$$

$$\sigma_{s_k} + \sigma_{s_p} + \sigma_{s_q} = 0$$

$$\left(\frac{\partial}{\partial t} + \nu k^2\right) b_{s_k}(\underline{k}) = \frac{1}{2} \sum_{\substack{\underline{k}+\underline{p}+\underline{q}=0 \\ \sigma_{s_k}+\sigma_{s_p}+\sigma_{s_q}=0}} C_{kpq}\, b^*_{s_p}(\underline{p})\, b^*_{s_q}(\underline{q})$$

$$\boxed{\begin{array}{c} \underline{k} + \underline{p} + \underline{q} = 0 \\ \sigma_{s_k} + \sigma_{s_p} + \sigma_{s_q} = 0 \end{array}}$$

$$\begin{cases} \sigma_{s_k} = 2s\, \underline{\Omega} \cdot \hat{\underline{k}} = 2s\, \Omega \cos\theta_k \\ \underline{k} \cdot \underline{\Omega} = k\, \Omega \cos\theta_k \end{cases}$$

$$k \cos\theta_k + p \cos\theta_p + q \cos\theta_q = 0$$

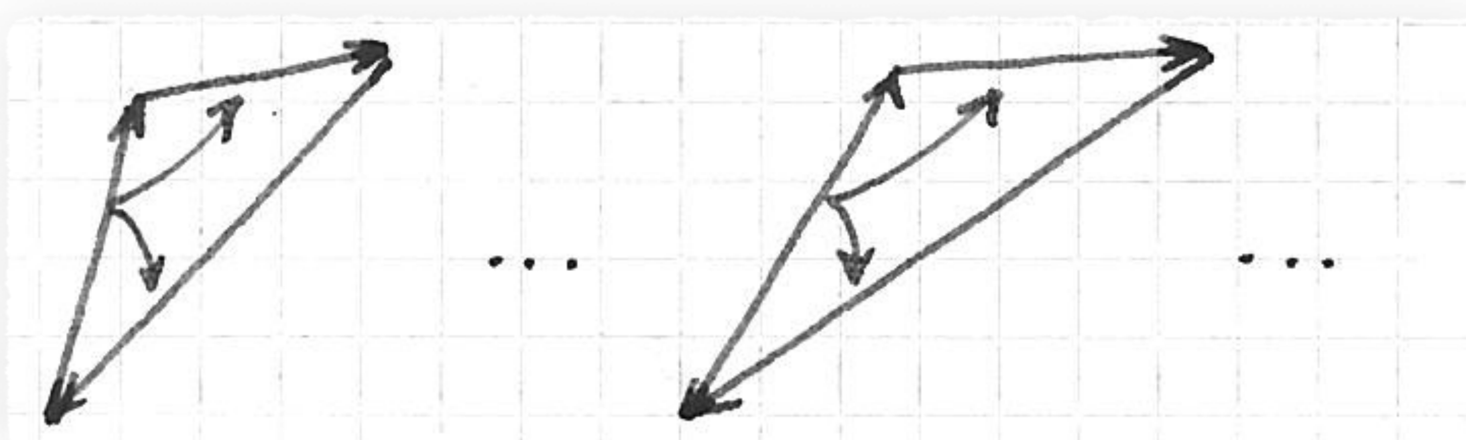
$$s_k \cos\theta_k + s_p \cos\theta_p + s_q \cos\theta_q = 0$$

$$\boxed{\frac{\cos\theta_k}{s_p q - s_q p} = \frac{\cos\theta_p}{s_q k - s_k q} = \frac{\cos\theta_q}{s_k p - s_p k}}$$

$$\cancel{s_p}\,\frac{\cos\theta_k}{q-p} = -\cancel{s_p}\,\frac{\cos\theta_p}{\frac{s_k}{s_p}q-k} = \cancel{s_p}\,\frac{\cos\theta_q}{\frac{s_k}{s_p}p-k}$$

$$\Rightarrow \quad \frac{|\cos\theta_k|}{q-p} = \frac{|\cos\theta_p|}{\left|\frac{s_k}{s_p}q-k\right|} = \frac{|\cos\theta_q|}{\left|\frac{s_k}{s_p}p-k\right|}$$

$$\Rightarrow \quad |\cos\theta_p| > |\cos\theta_k|,\ |\cos\theta_q|$$



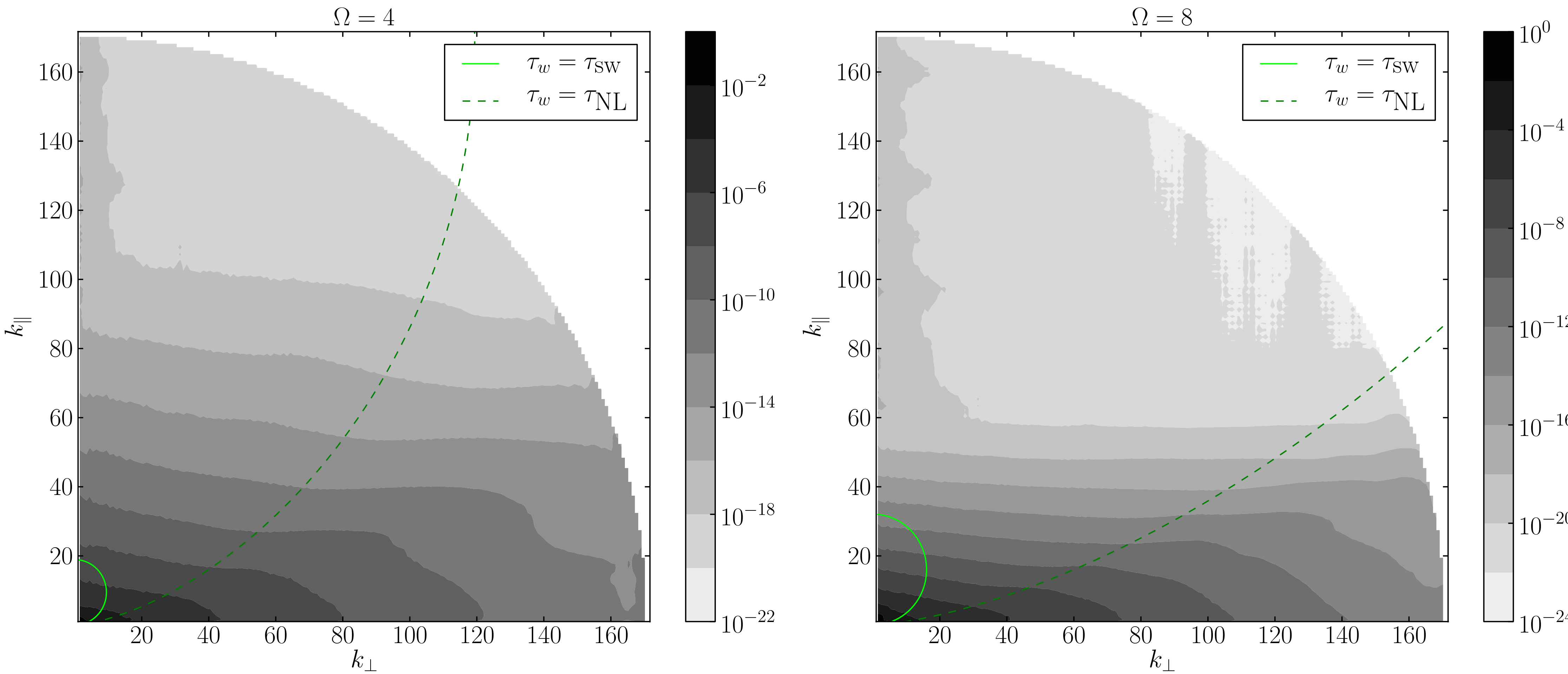


FIG. 2. Isocontours of the axisymmetric energy spectrum $e(k_\perp, k_\parallel)/\sin(\theta_k)$ in the runs with $\Omega = 4$ (above) and 8 (below); dark means larger energy density (in logarithmic scale). Lines indicating the modes for which the wave time becomes equal to the sweeping time, and to the turnover time, are given as references. It should be noted that the energy does not accumulate near the modes with $\tau_\omega = \tau_{\rm NL}$, unlike what is expected in theories dealing with the concept of critical balance.[24]

Clark Di Leoni & Mininni, PRE (2015)

Phenomenology:

$$\frac{dE}{dt} \sim \epsilon \sim \frac{\delta v_\ell^2}{\tau}$$

$$\epsilon \sim \frac{\delta v_\ell^2}{t_\ell} f\left(\frac{t_\ell}{t_\Omega}\right)$$

$$t_\ell \sim \frac{\ell_\perp}{\delta v_\ell}$$

$$\epsilon \sim \frac{\delta v_\ell^3}{\ell_\perp} f\left(\frac{t_\ell}{t_\Omega}\right)$$

Phenomenology:

$$\mathcal{P}\left(\frac{t_\ell}{t_\Omega}\right) \sim \frac{t_\Omega}{t_\ell}$$

$$\epsilon \sim \frac{\delta v_\ell^3}{\ell_\perp} \frac{t_\Omega}{\ell_\perp} \delta v_\ell \sim \frac{\delta v_\ell^4}{\ell_\perp^2} \frac{1}{\Omega}$$

$$\delta v_\ell^2 \sim \Omega^{1/2} \epsilon^{1/2} \ell_\perp$$

$$E(k_\perp) \sim \frac{\delta v_\ell^2}{k_\perp} \sim \Omega^{1/2} \epsilon^{1/2} k_\perp^{-2}$$

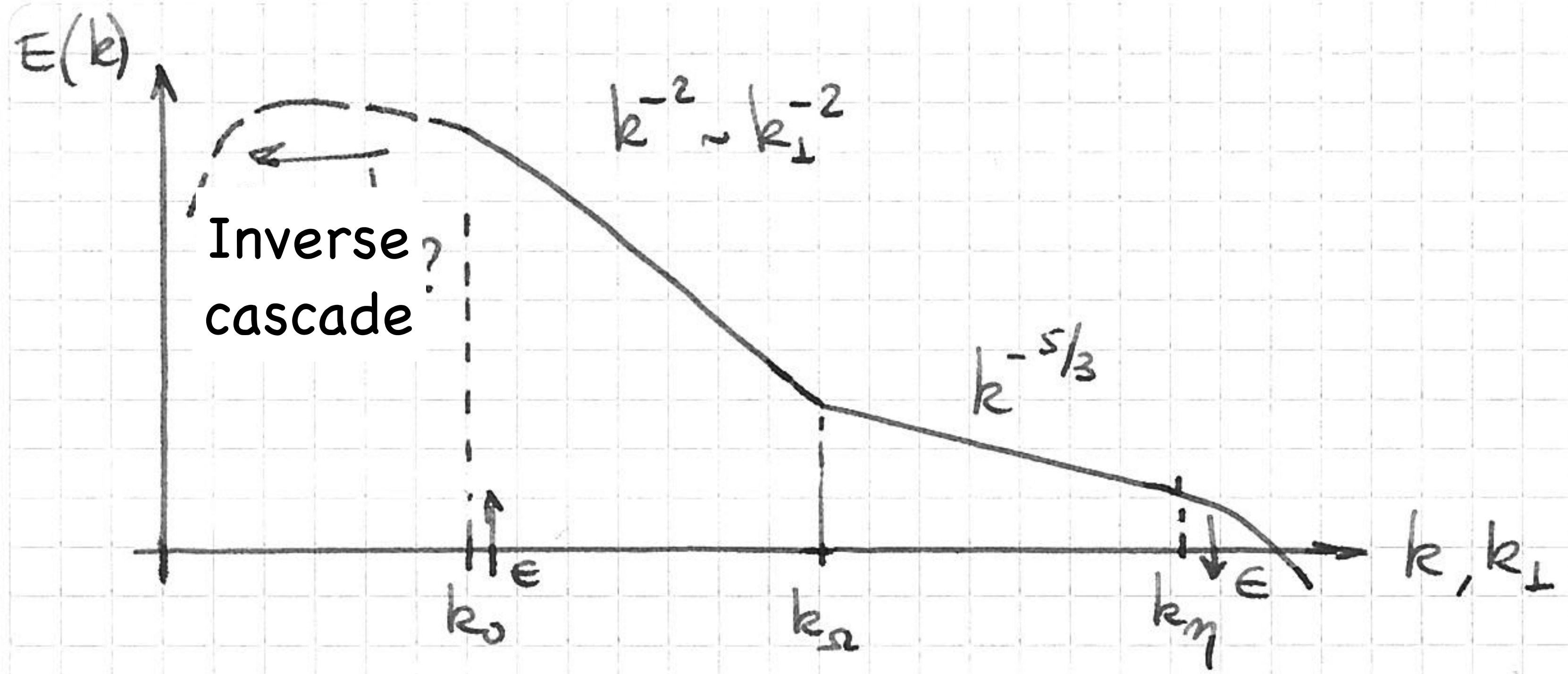
$$\Rightarrow \boxed{E(k_\perp) = C\,\Omega^{1/2} \epsilon^{1/2} k_\perp^{-2}}$$

$$t_\ell \sim t_{\sigma_k}$$

$$t_\ell \sim \frac{\ell}{\delta v_\ell} \sim \frac{1}{\Omega}$$

$$\Rightarrow \quad \Omega \ell \sim \Omega^{1/4} \epsilon^{1/4} \ell^{1/2}$$

$$\ell^{1/2} \sim \epsilon^{1/4} \Omega^{-3/4} \sim \left(\frac{\epsilon}{\Omega^3}\right)^{1/4}$$



$$\Rightarrow \quad \boxed{k_\Omega \sim \sqrt{\frac{\Omega^3}{\epsilon}}}$$

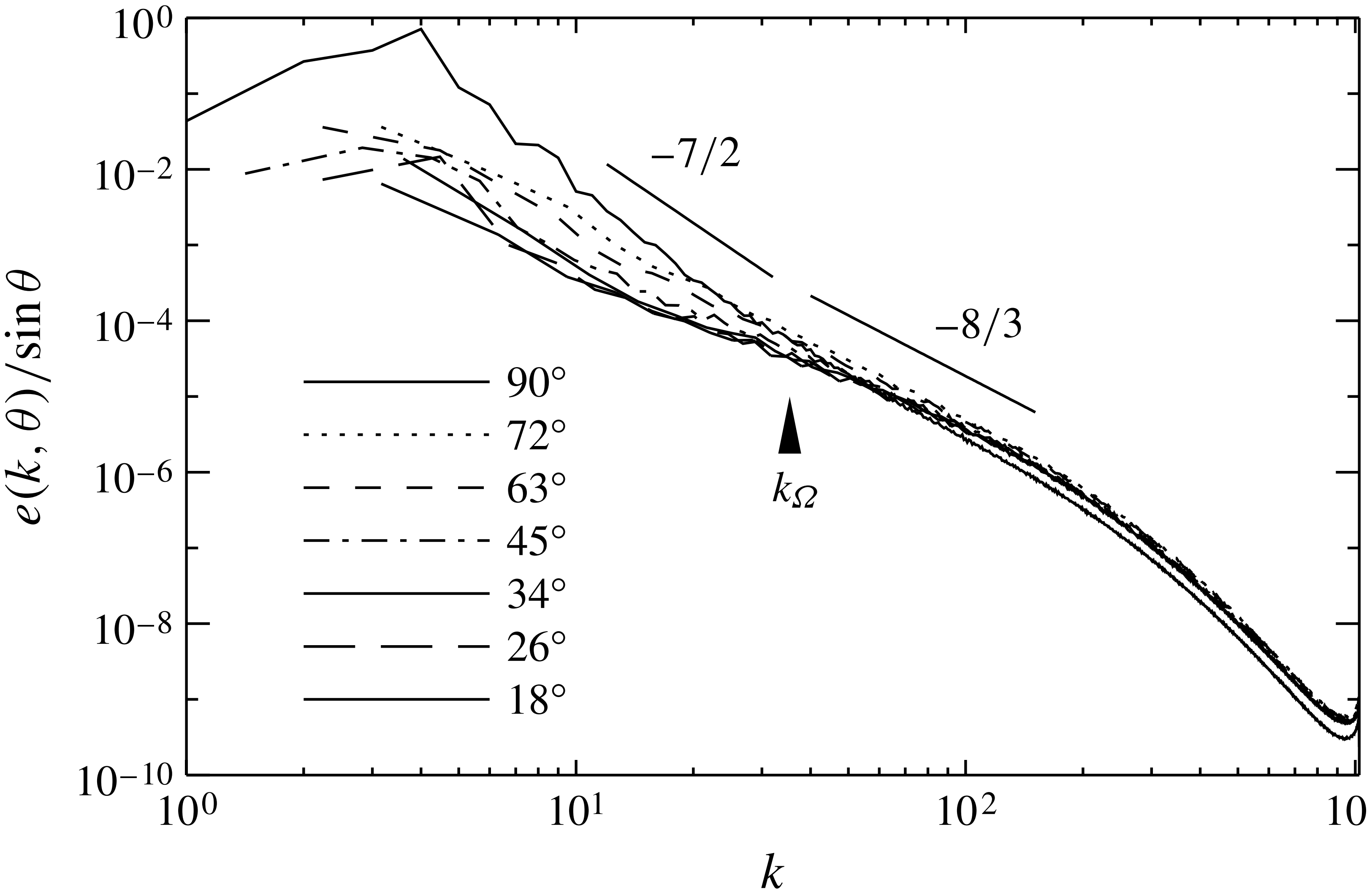


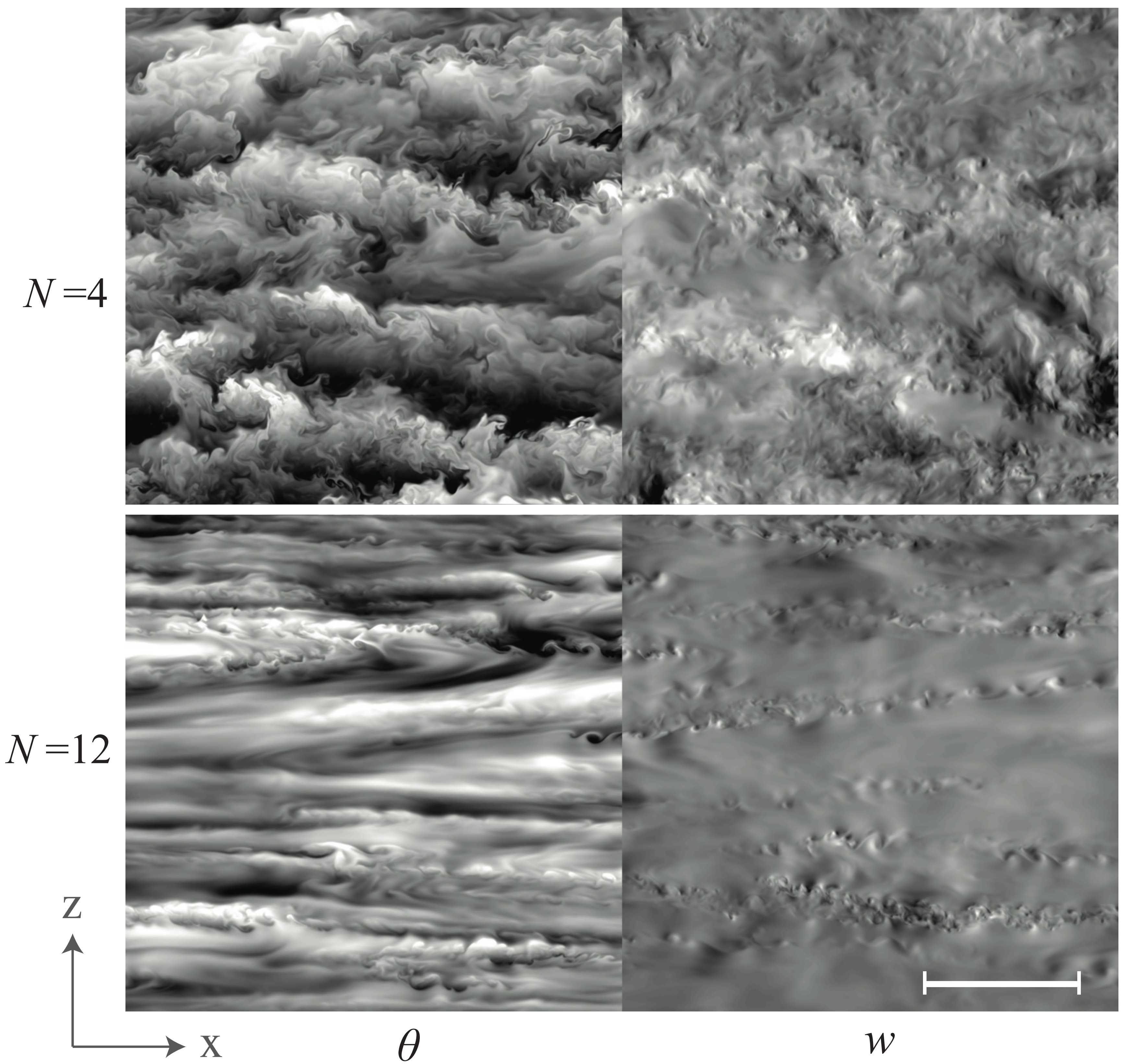
FIGURE 5. Angular distribution of energy spectra for different co-latitudes θ, averaged for $5 \leqslant t \leqslant 6$; the Zeman wavenumber is indicated with an arrow. Slopes $-7/2$ (corresponding to $\sim k^{-5/2}$ scaling in units of the reduced energy spectra) and $-8/3$ (corresponding to Kolmogorov $\sim k^{-5/3}$ scaling) are shown as a reference. Note the recovery of isotropy beyond k_{Ω}.

Mininni et al., JFM (2012)

STABLY STRATIFIED TURBULENCE

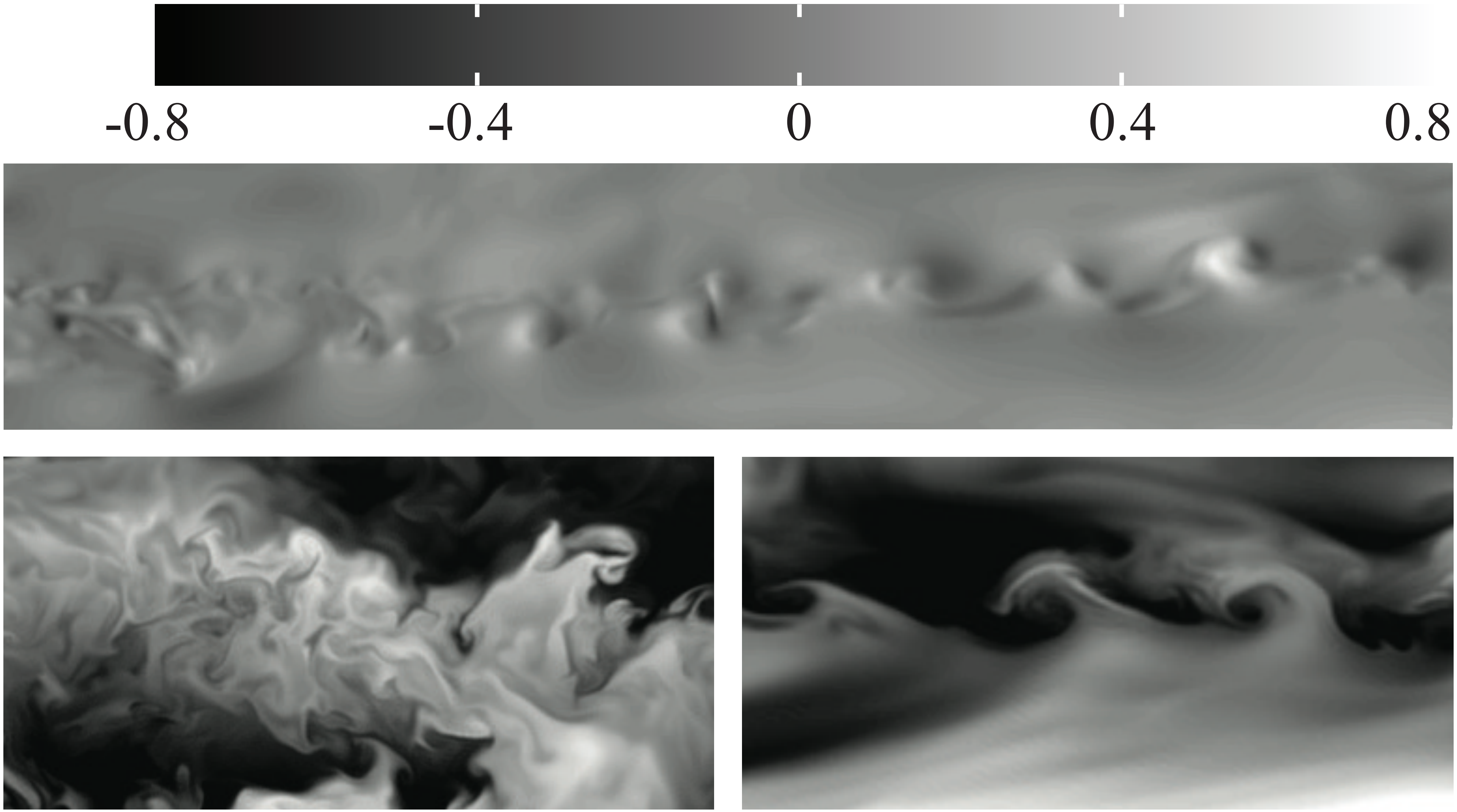
$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p - N\theta\hat{z} + \nu\nabla^2\mathbf{u} + \mathbf{F},$$

$$\partial_t \theta + \mathbf{u} \cdot \nabla\theta = N u_z + \kappa\nabla^2\theta,$$

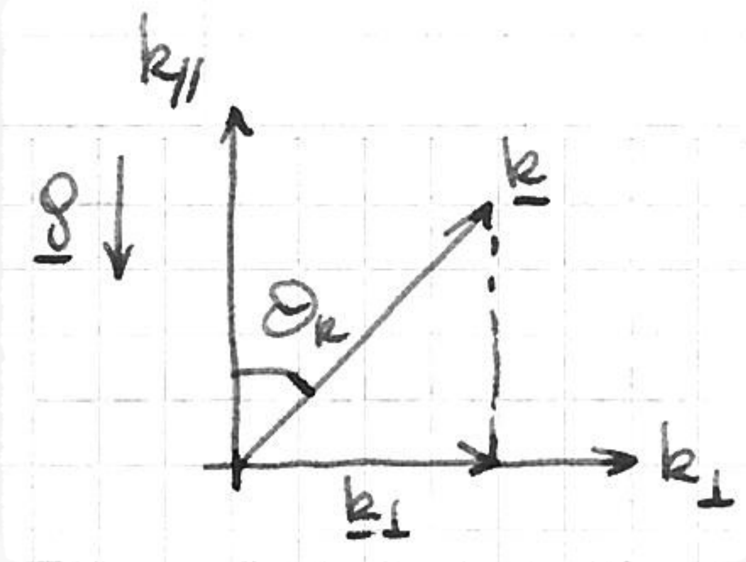


2048^3
Rorai et al., PRE (2014)

STABLY STRATIFIED TURBULENCE

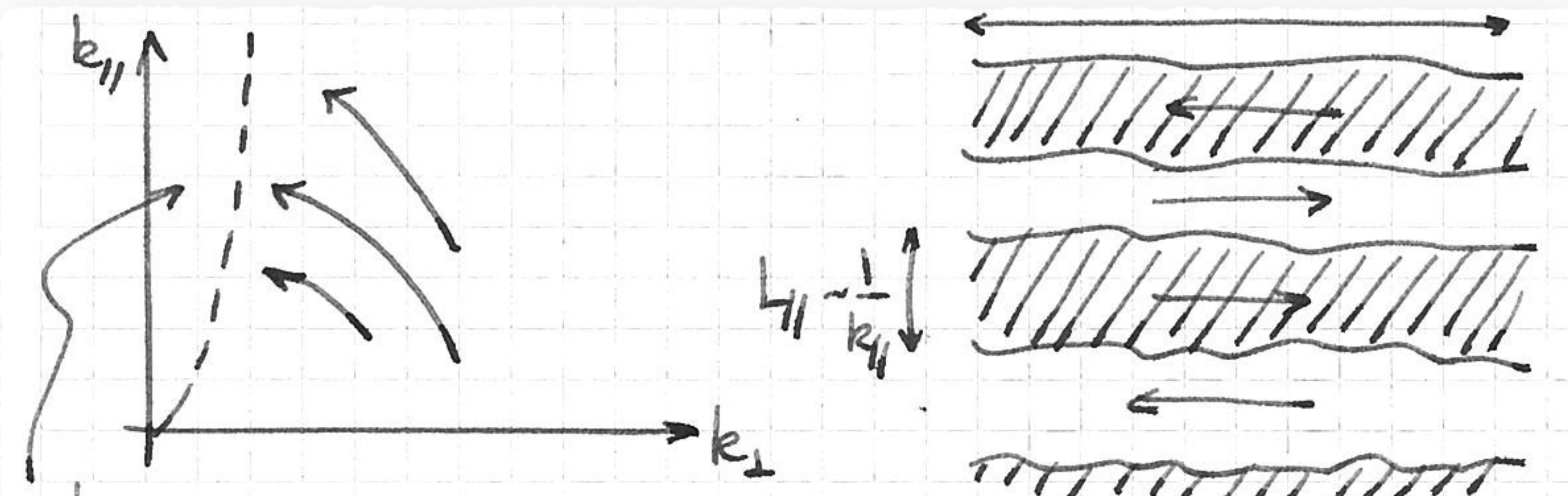


Stratified turbulence



$$\sigma_{s_k} = s_k N \frac{|\underline{k}_\perp|}{|\underline{k}|} = s_k N \,\mathrm{sen}\,\theta_k$$

$$\sigma_{s_k} + \sigma_{s_p} + \sigma_{s_q} = 0 = N\left(s_k \,\mathrm{sen}\,\theta_k + s_p \,\mathrm{sen}\,\theta_p + s_q \,\mathrm{sen}\,\theta_q\right)$$



Slow modes

$$Fr_{\parallel} = \frac{U_\perp}{NL_\parallel} \quad , \quad Fr_\perp = \frac{U_\perp}{NL_\perp}$$

$$\frac{U_\perp}{L_\perp} \approx \frac{U_\parallel}{L_\parallel} \quad \Longrightarrow \quad U_\parallel \approx \frac{L_\parallel}{L_\perp} U_\perp \ll U_\perp$$

$$\begin{cases} \dfrac{\partial \underline{u}}{\partial t} + \underline{u} \cdot \underline{\nabla} \underline{u} = -\dfrac{1}{\rho_0} \underline{\nabla} p - N\theta \hat{z} + \nu \nabla^2 \underline{u} & (1) \\ \dfrac{\partial \theta}{\partial t} + \underline{u} \cdot \underline{\nabla} \theta = Nw + \kappa \nabla^2 \theta & (2) \end{cases}$$

$$\underbrace{\frac{\partial \underline{\omega}}{\partial t}}_{\sim\ \frac{\omega_\perp}{\tau_L}} + \underbrace{\underline{u}\cdot\underline{\nabla}\,\underline{\omega}}_{\sim\ \frac{U_\parallel}{L_\parallel}\omega_\perp} = \underbrace{\underline{\omega}\cdot\underline{\nabla}\,\underline{u}}_{\sim\ \frac{\omega_\parallel}{L_\parallel}U_\perp} - \underbrace{N\underline{\nabla}\theta\times\hat{z}}_{\sim\ \frac{N\theta}{L_\perp}} + \underbrace{\nu\nabla^2\underline{\omega}}_{\sim\ \cancel{\nu\frac{\omega_\perp}{L_\parallel^2}}}$$

$Re \gg 1$

$$\omega_\perp \frac{U_\perp}{L_\perp} \sim \omega_\perp \frac{U_\parallel}{L_\parallel}$$

$$y \begin{cases} \omega_\perp \sim \dfrac{U_\perp}{L_\parallel} \\[2ex] \omega_\parallel \sim \dfrac{U_\perp}{L_\perp} \end{cases}$$

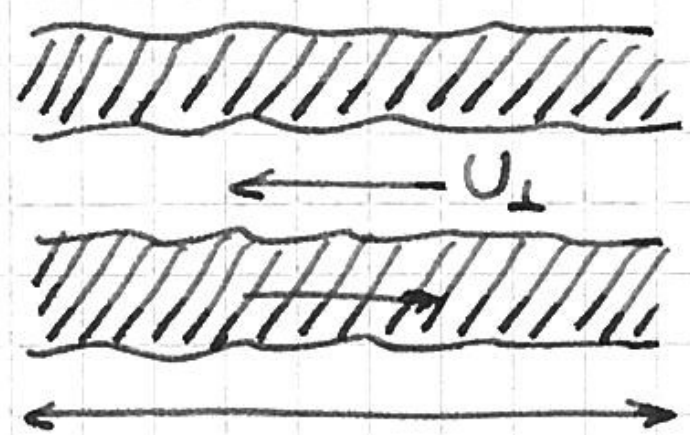
$$\Rightarrow \quad \frac{U_\perp^2}{L_\parallel L_\perp} \sim \frac{N\theta}{L_\perp}$$

$$\frac{U_\parallel}{L_\parallel}\theta \sim \frac{U_\perp}{L_\perp}\theta \sim N U_\parallel$$

$$\Rightarrow \quad \frac{U_\perp^2}{L_\parallel L_\perp} \sim N^2 \frac{U_\parallel}{U_\perp}$$

$$\Rightarrow \quad Fr_\parallel^2 = \frac{U_\perp^2}{N^2 L_\parallel^2} \sim \frac{1}{\cancel{N^2 L_\parallel^2}} \cancel{N^2}\cancel{L_\parallel} L_\perp \frac{U_\parallel}{U_\perp} \sim \frac{L_\perp}{L_\parallel}\frac{U_\parallel}{U_\perp} \sim 1$$

$$\Rightarrow \quad \boxed{Fr_\parallel \sim 1} \quad \text{y} \quad \boxed{L_\parallel \sim \frac{U_\perp}{N}}$$



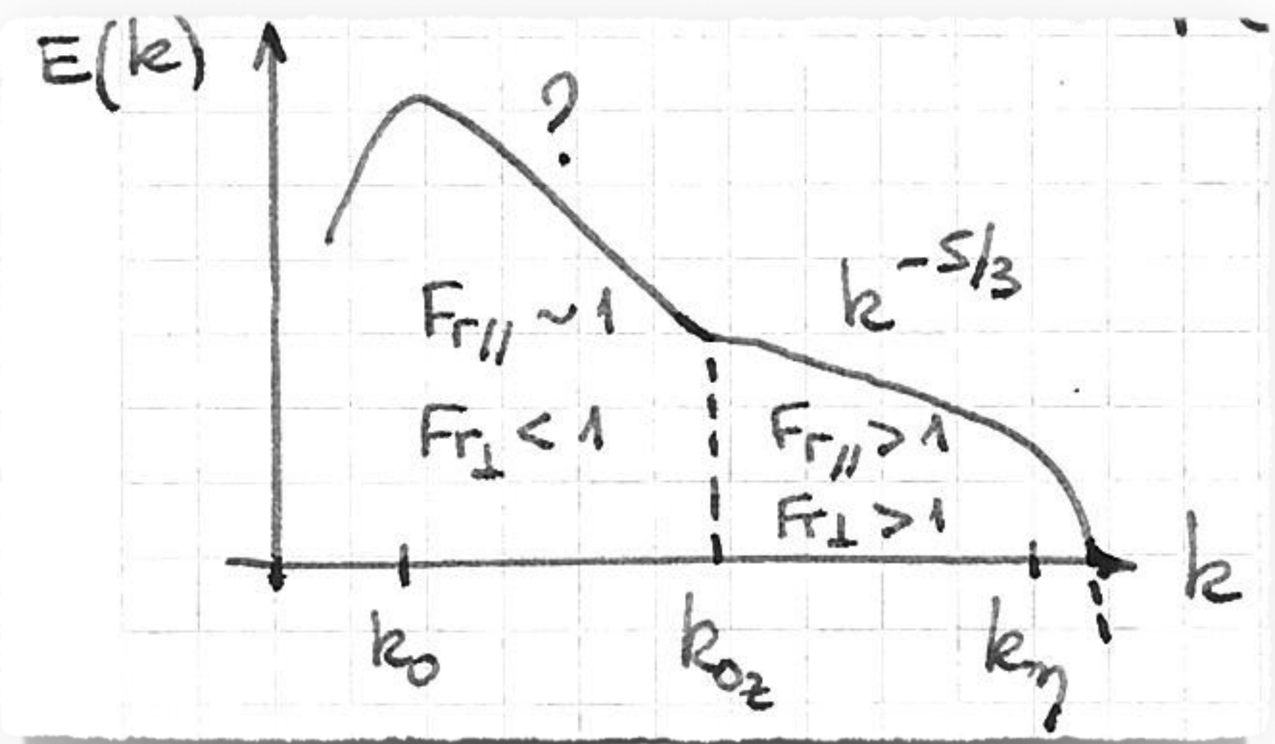
$$Fr_\perp = \frac{U_\perp}{N L_\perp} \sim \frac{L_\parallel}{L_\perp}$$

$$\frac{|\underline{\omega}\cdot\underline{\nabla}\underline{v}|}{|\nu\nabla^2\underline{\omega}|} \sim \underbrace{\frac{\omega_{\parallel}}{L_{\parallel}}U_{\perp}}_{\frac{U_{\perp}^2}{L_{\parallel}L_{\perp}}} \cdot \frac{L_{\parallel}^2}{\nu\,\omega_{\perp}} \sim \frac{U_{\perp}}{\nu L_{\perp}}\,L_{\parallel}^2$$

$$\frac{1}{\omega_{\perp}} \sim \frac{L_{\parallel}}{U_{\perp}}$$

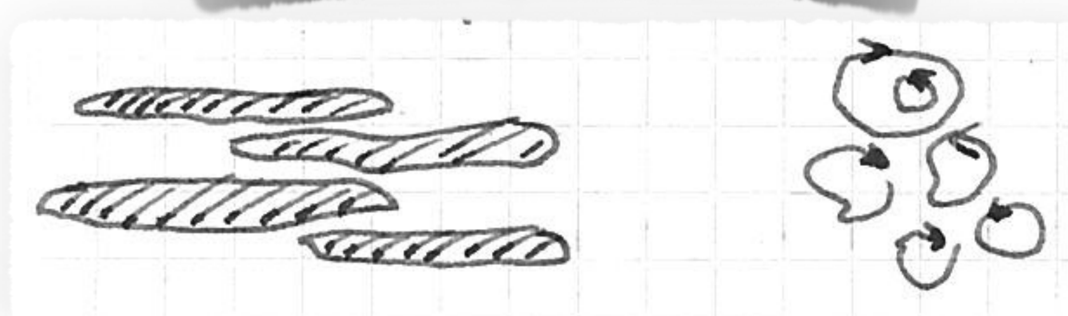
$$\sim \underbrace{\frac{U_{\perp}L_{\perp}}{\nu}}_{Re}\,\underbrace{\frac{L_{\parallel}^2}{L_{\perp}^2}}_{Fr_{\perp}^2} \sim Re\,Fr_{\perp}^2 = \boxed{R_B \gg 1}$$

$$\Longrightarrow \boxed{Re \gg \frac{1}{Fr_{\perp}^2}}$$



$$\boxed{L_{oz} \sim \left(\frac{\epsilon}{N^3}\right)^{1/2}}$$

$$k_{oz} \sim \sqrt{\frac{N^3}{\epsilon}}$$



Particles → What are the relevant equations of motion in geophysics?

→ What is the right description?

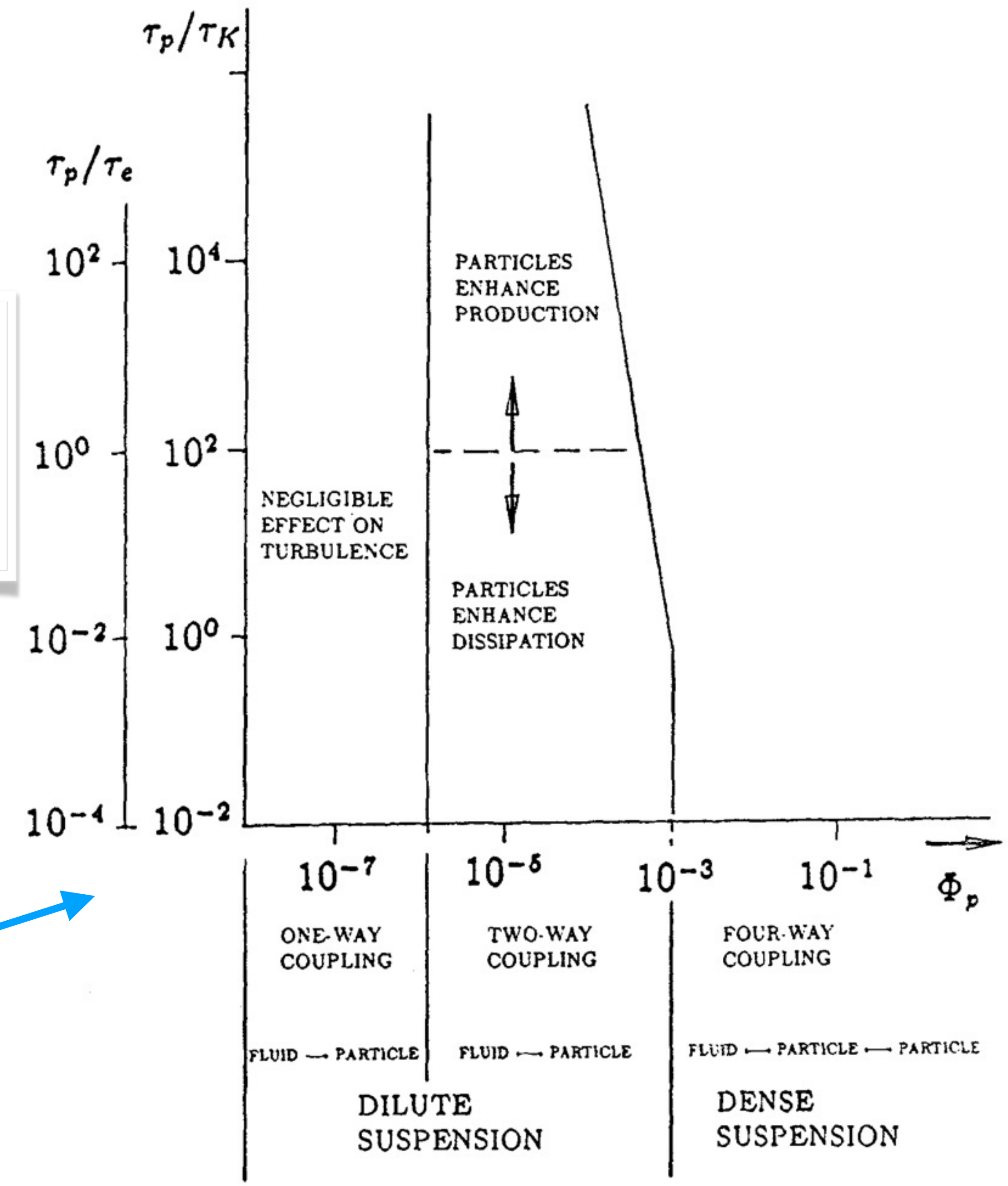
→ Clustering of inertial particles

+ Bec, Gustavsson & Mehlig 2024

Particles in turbulent flows

Maxey and Riley (1983):

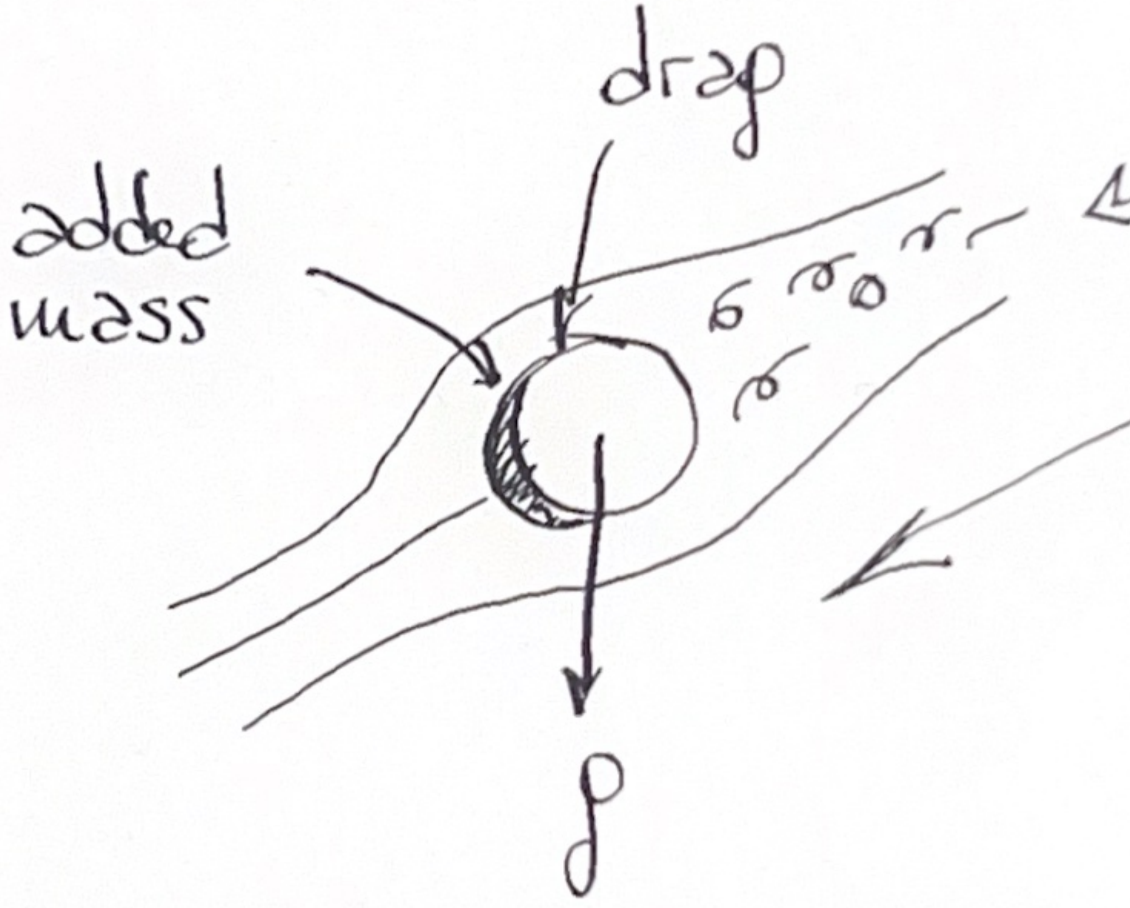
$$\mathbf{F}_{\mathrm{h}} = \frac{4\pi}{3}\varrho_{\mathrm{f}}a^3\frac{\mathrm{D}}{\mathrm{D}t}\mathbf{u}(\mathbf{x},t) - 6\pi\nu\varrho_{\mathrm{f}}a\left[\mathbf{v}-\mathbf{u}(\mathbf{x},t)\right] - \frac{4\pi}{3}\varrho_{\mathrm{f}}a^3\,\mathbf{g}$$
$$-\frac{2\pi}{3}\varrho_{\mathrm{f}}a^3\frac{\mathrm{d}}{\mathrm{d}t}\left[\mathbf{v}-\mathbf{u}(\mathbf{x},t)\right] - 6\sqrt{\pi\nu}\,\varrho_{\mathrm{f}}a^2\int_0^t\frac{\mathrm{d}s}{\sqrt{t-s}}\frac{\mathrm{d}}{\mathrm{d}s}\left[\mathbf{v}-\mathbf{u}(\mathbf{x},s)\right].$$



Coupling (see Elghobashi, 1994)

$$\rho_p \frac{d\underline{v}}{dt} = \frac{4\pi}{3}\rho_f a^3 \frac{D\underline{u}}{Dt} \overbrace{- 6\pi\nu\rho_f a(\underline{v}-\underline{u})}^{\text{drag}} - \frac{4\pi}{3}\rho_f a^3 \underline{g}$$

$$- \frac{2\pi}{3}\rho_f a^3 \frac{d}{dt}(\underline{v}-\underline{u}) - 6\sqrt{\pi\nu}\,\rho_f a^2 \int_0^t \frac{ds}{\sqrt{t-s}} \frac{d}{ds}(\underline{v}-\underline{u})$$



For very small a, $\rho_p \gg \rho_f$, $Re_p = \frac{a|\underline{u}-\underline{v}|}{\nu} \ll 1$

$$\frac{d\underline{v}}{dt} = \frac{-1}{\tau_p}(\underline{v}-\underline{u}) + \underline{g}$$

with $\quad \tau_p = \frac{m_p}{6\pi\nu\rho_f a} \qquad m_p = \rho_p V_p$

If $\underline{u}=0 \implies \cancel{\frac{d\underline{v}}{dt}} = -\frac{1}{\tau_p}\underline{v} + \underline{g} = 0$

$$\implies \underline{v} = \boxed{\tau_p \underline{g} = \underline{W}}$$

If $|\tau_p \underline{g}| \ll |\underline{u}| \implies \boxed{\frac{d\underline{v}}{dt} \approx -\frac{1}{\tau_p}(\underline{v}-\underline{u})}$

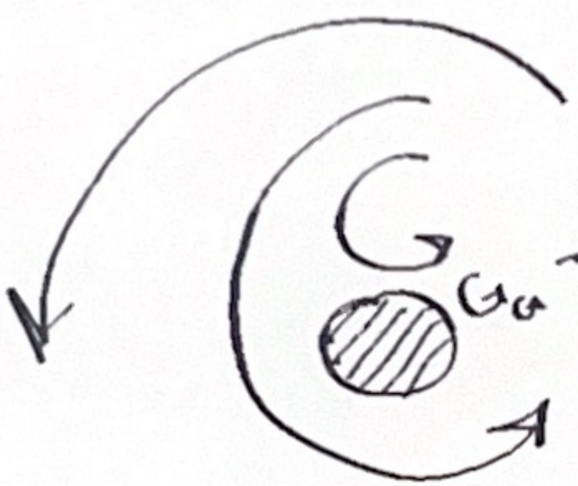
From
$$\dot{\underline{v}} = -\frac{1}{\tau_p}(\underline{v}-u)$$

we can define a Stokes number

$$St = \frac{\tau_p}{\tau_f}$$

Usually $\tau_f = \tau_\eta = \left(\nu/\varepsilon\right)^{1/2}$

But there isn't an agreement on this!



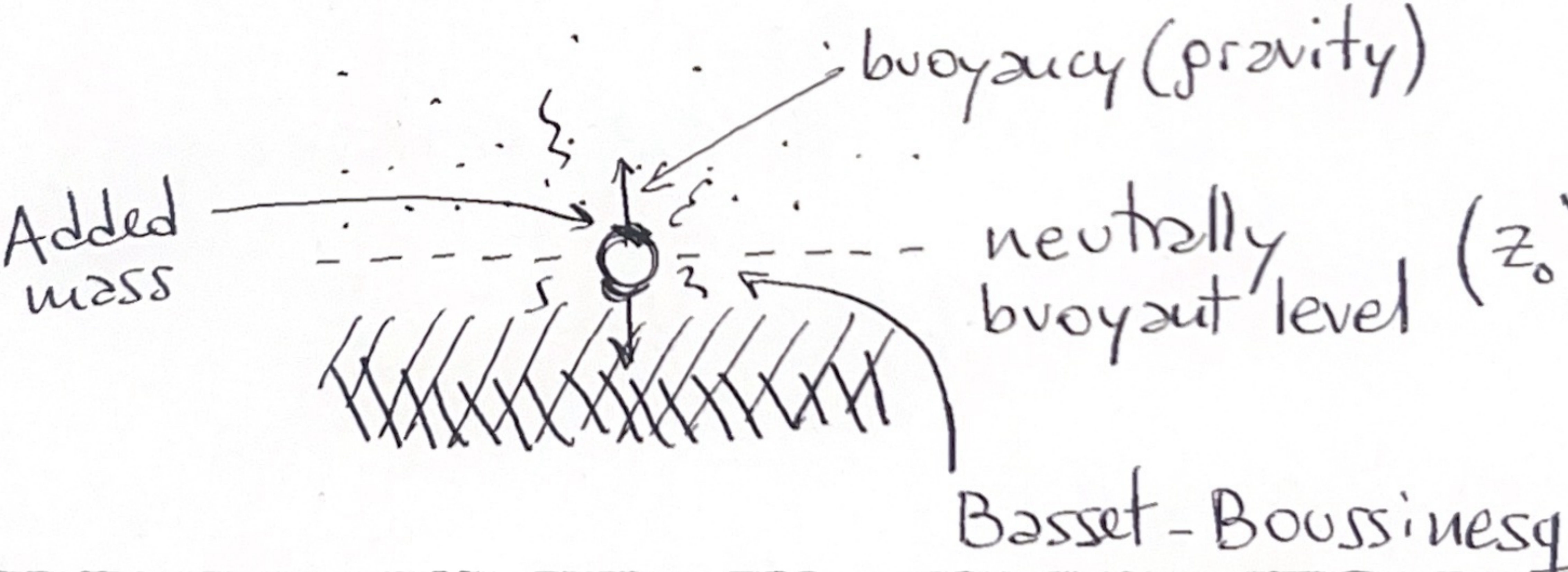
small eddies won't affect the particle

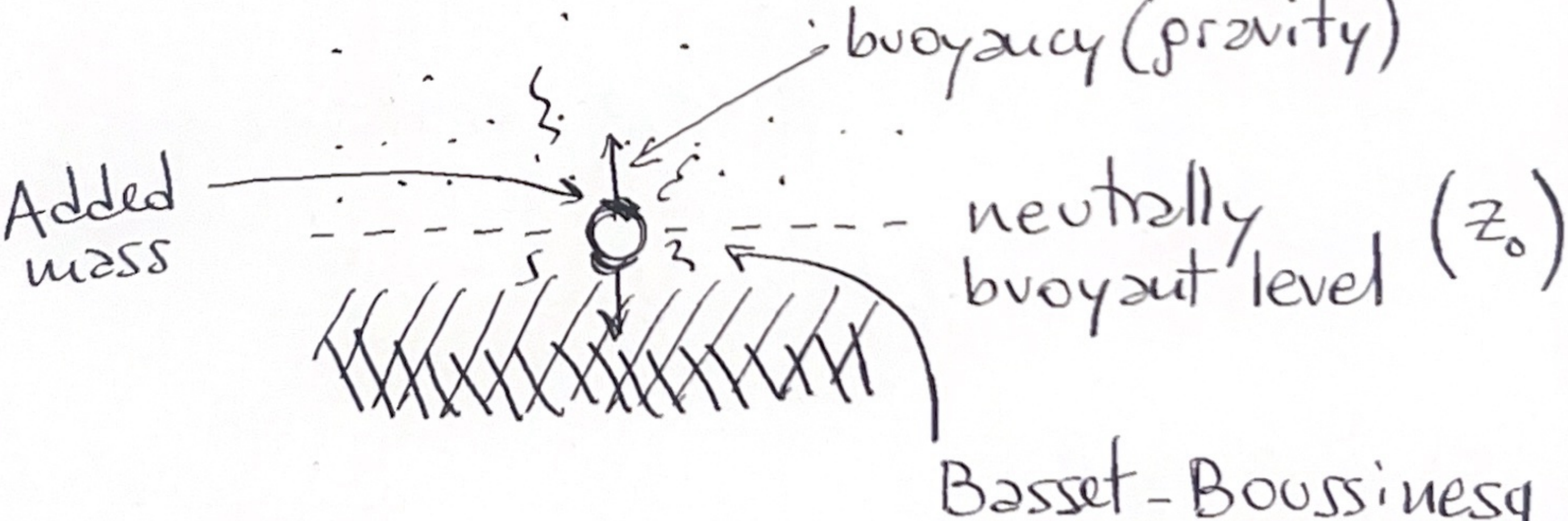
Note $\dot{\underline{v}} = -\frac{1}{\tau_p}(\underline{v}-\underline{u})$ is a low passband filter

Let's keep using MRG:

In a stratified flow:

$$\dot{\underline{v}} = \frac{1}{\tau_p}(\underline{u}-\underline{v}) - \frac{2N}{3}\left[N(z-z_0)-\xi\right]\hat{z} + \frac{D\underline{u}}{Dt} + \sqrt{\frac{3}{\pi\tau_p}}\int^t d\tau \frac{\frac{d}{d\tau}(\underline{u}-\underline{v})}{\sqrt{t-\tau}}$$





under certain conditions this equation can be rewritten as

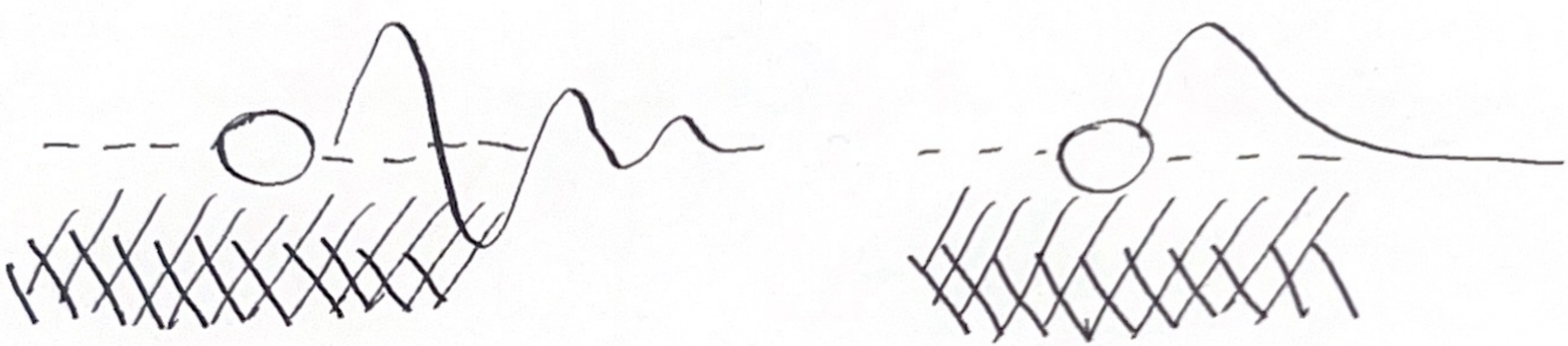
$$\left(\ddot{z}_p - \ddot{z}_f\right) + \frac{1}{\tau_p}\left(\dot{z}_p - \dot{z}_f\right) + \frac{2}{3}N^2\left(z_p - z_f\right) = \underline{F}_{BB} \cdot \hat{z}$$

which is the equation for the forced damped oscillator.

For $E_{BB} = 0$, we can get

+ overdamped
+ critically damped
+ underdamped

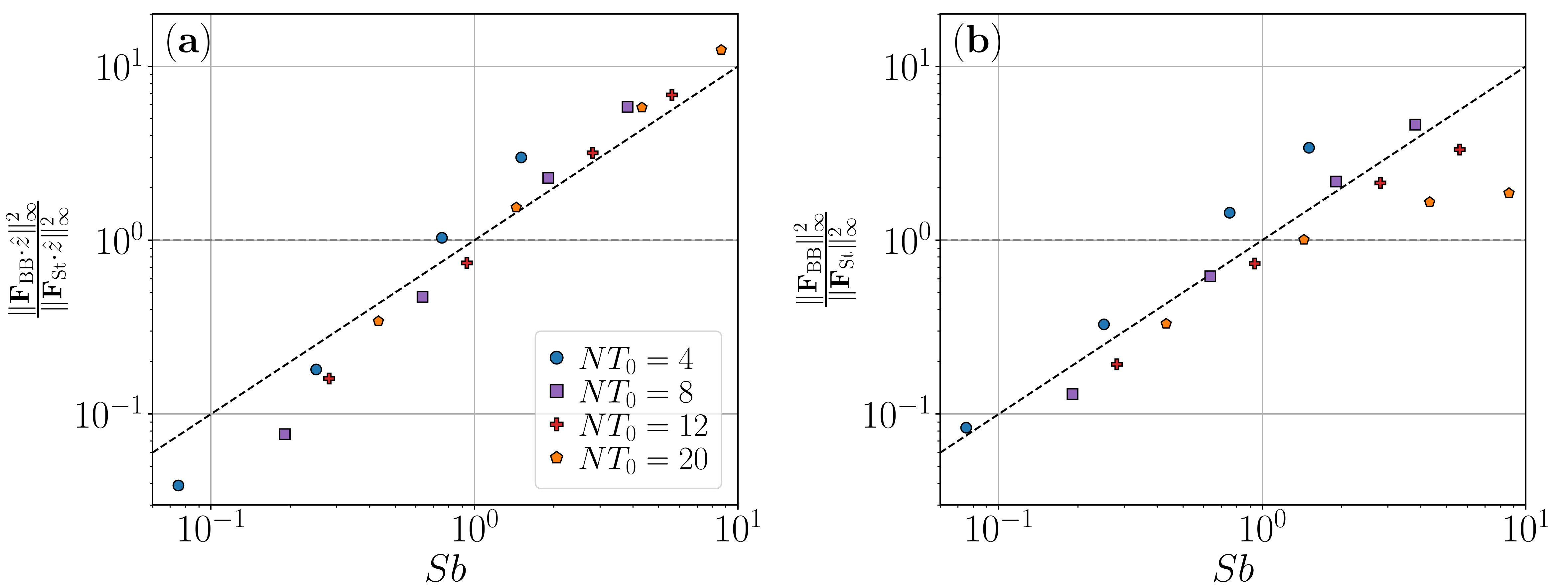
oscillations depending on τ_p:



Using these solutions, we can bound

$$\frac{\| \underline{F}_{BB} \cdot \hat{z} \|_\infty^2}{\| \underline{F}_{St} \cdot \hat{z} \|_\infty^2} \leq 16 N \tau_p$$

We can then define $\boxed{S_b = N \tau_p}$



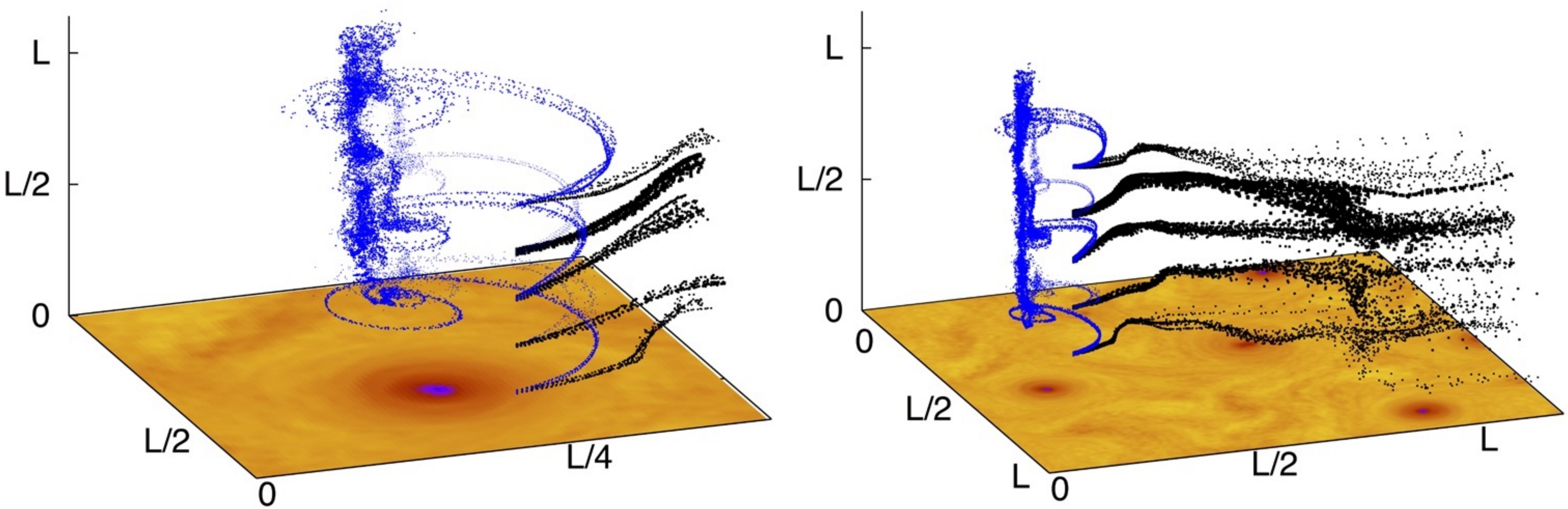
Reartes & Mininni (2024)

In a rotating flow:

$$\dot{\underline{v}} = \frac{1}{\tau_p}(\underline{u}-\underline{v}) - \left(1-\frac{\rho_f}{\rho_p}\right) g\hat{z} + \frac{3\rho_f}{\rho_f+2\rho_p}\frac{D\underline{u}}{Dt}$$

$$-2\underline{\Omega}\times\left(\underline{v}-\frac{3\rho_f}{\rho_f+2\rho_p}\underline{u}\right)$$

$$-\left(1-\frac{3\rho_f}{\rho_f+2\rho_p}\right)\underline{\Omega}\times\underline{\Omega}\times(\underline{x}-\underline{x}_0)$$



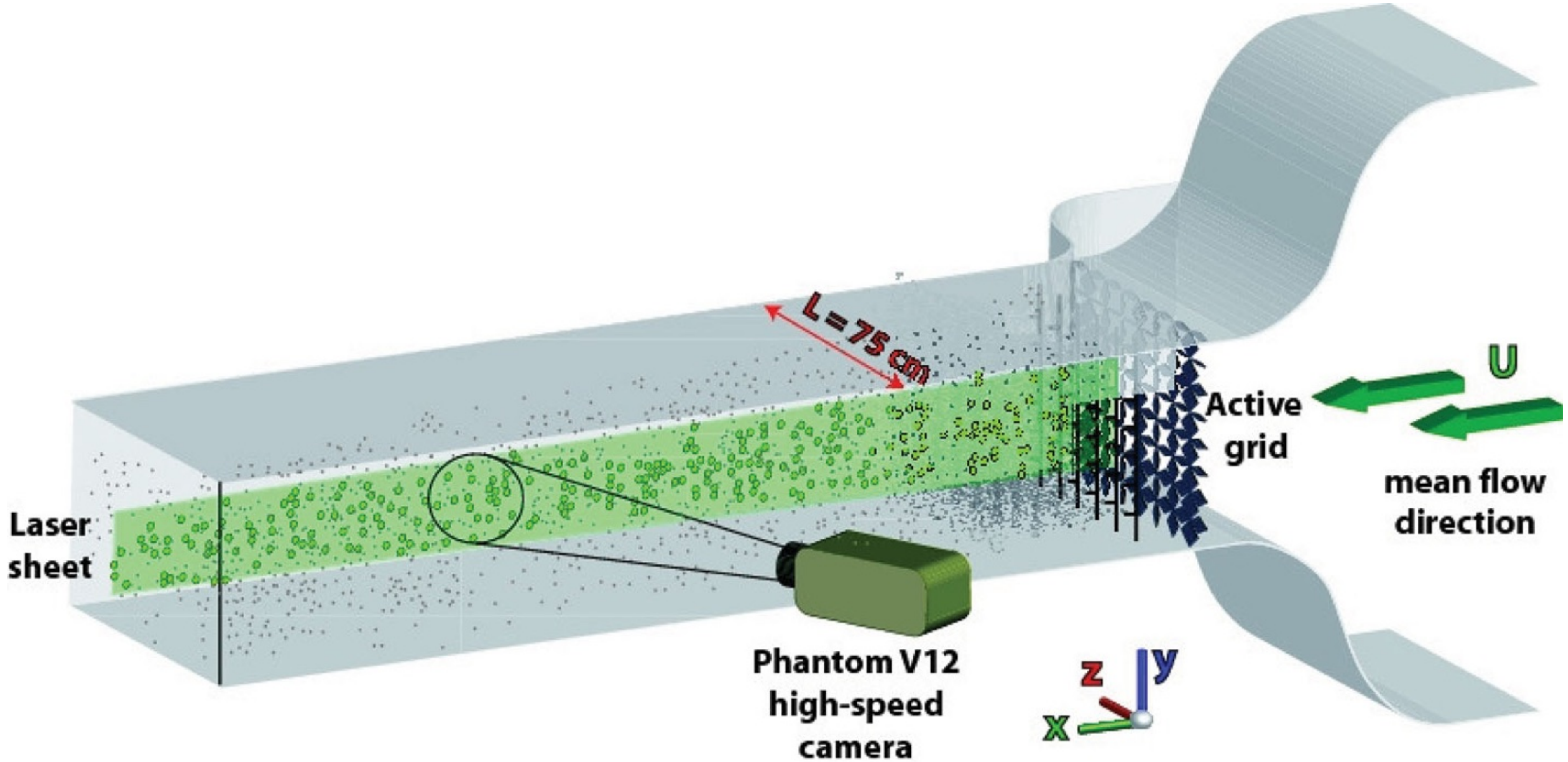
Biferale et al. (2016)

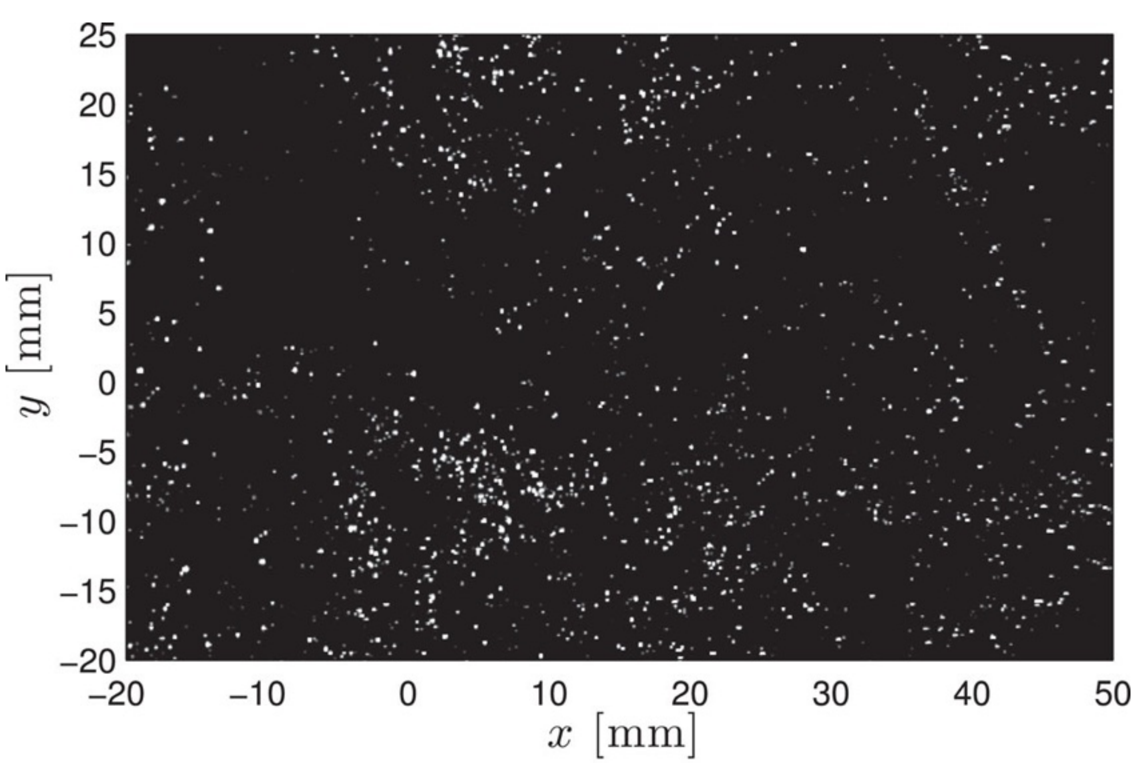
Clustering

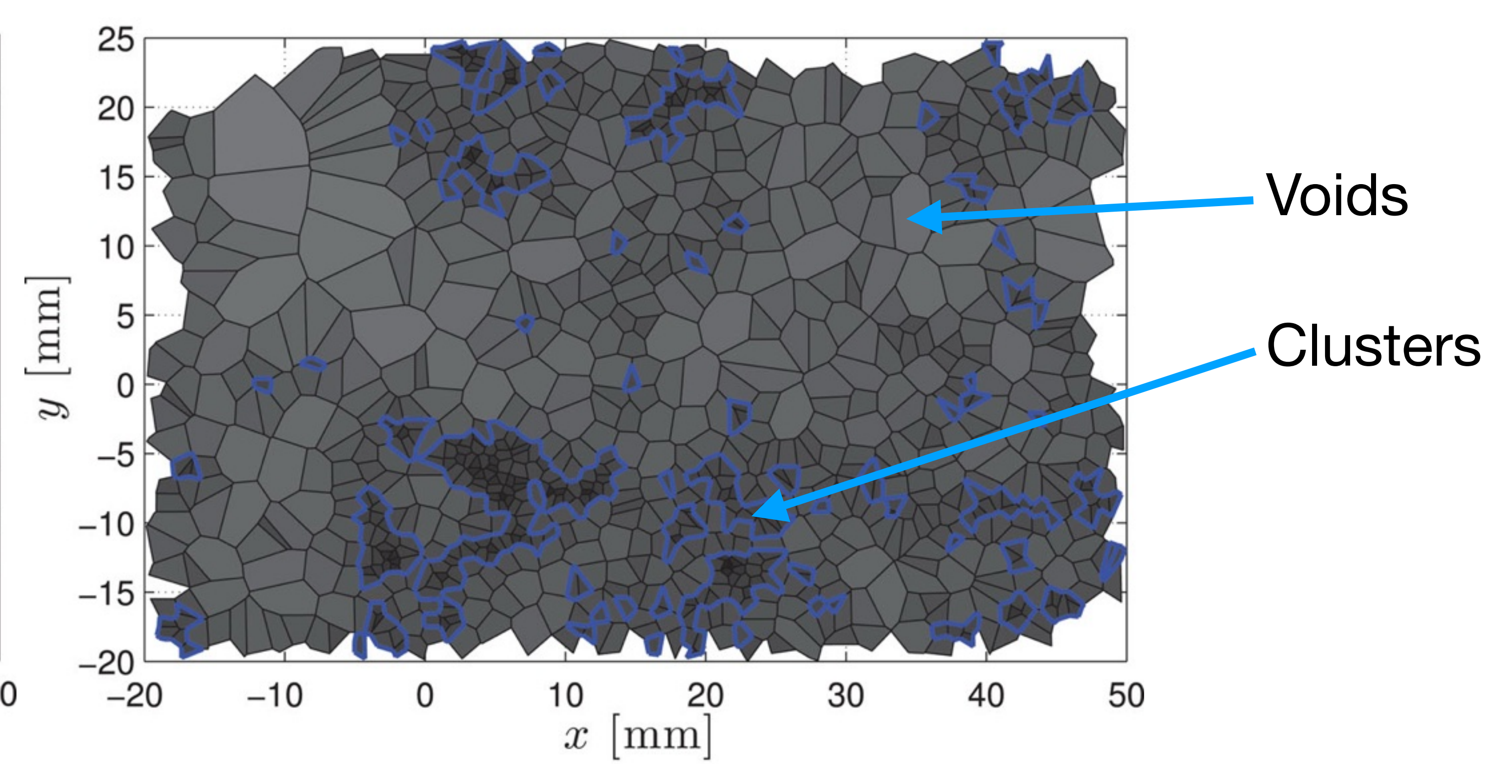
Maxey's centrifuge
Maxey 1987
Balkovsky-Falkovich-
Fouxon 2001

Sweep-stick mechanism
Goto & Vassilicos 2008

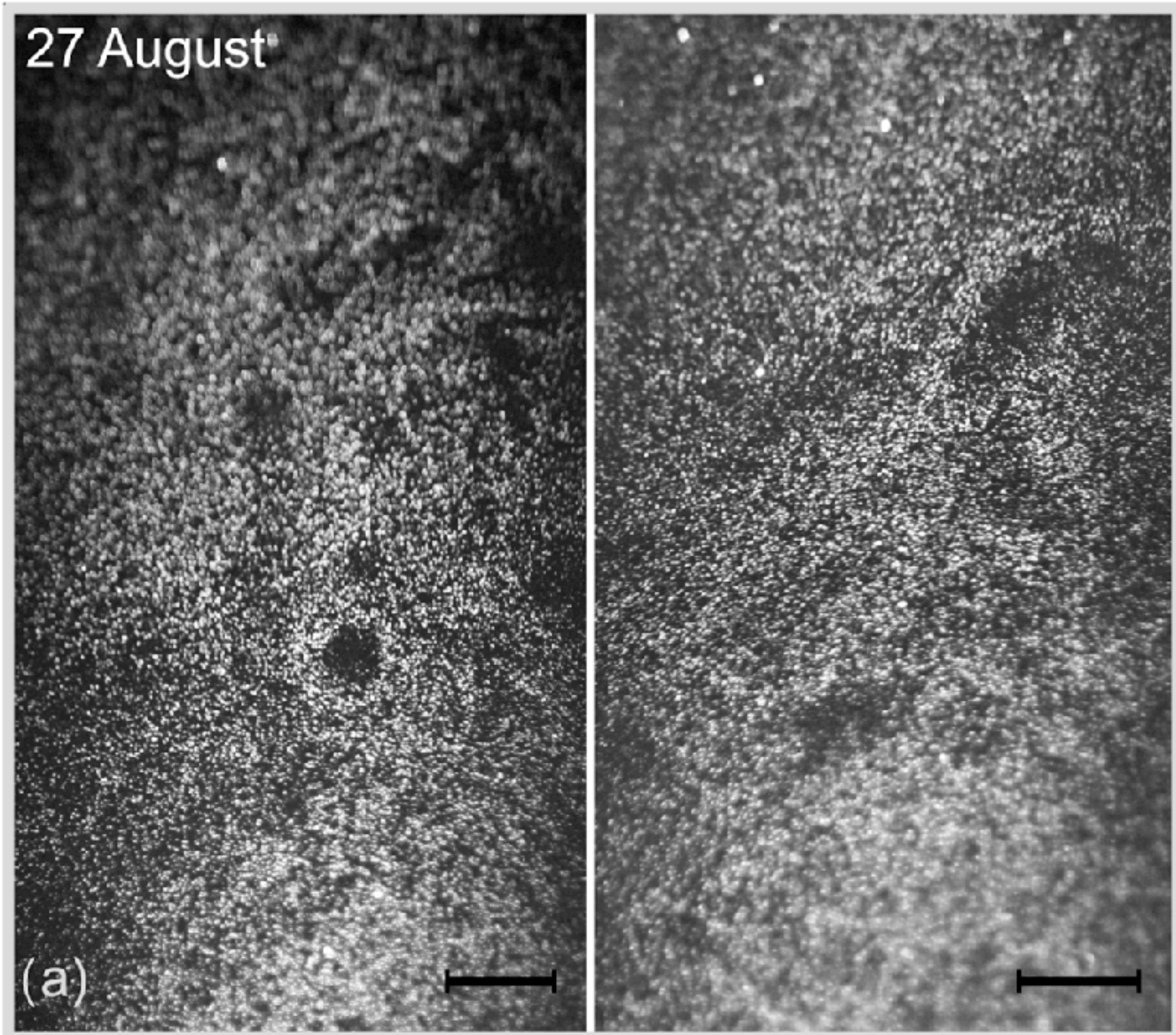
Other multiscale/large-scale
mechanisms (see Bec, Gustavsson,
Mehlig 2024)







Obligado et al. (2014)



Karpińska et al. (2019)

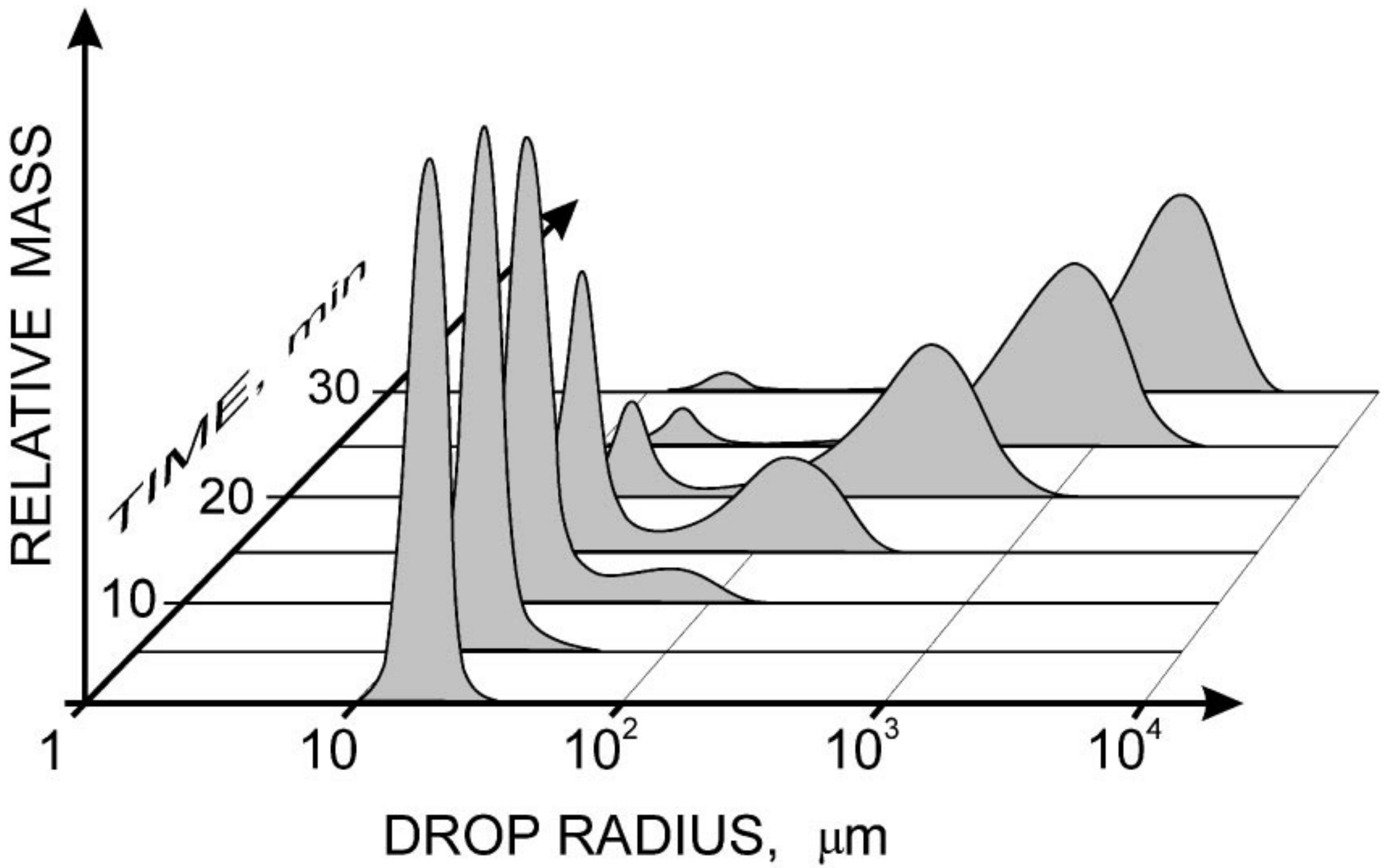
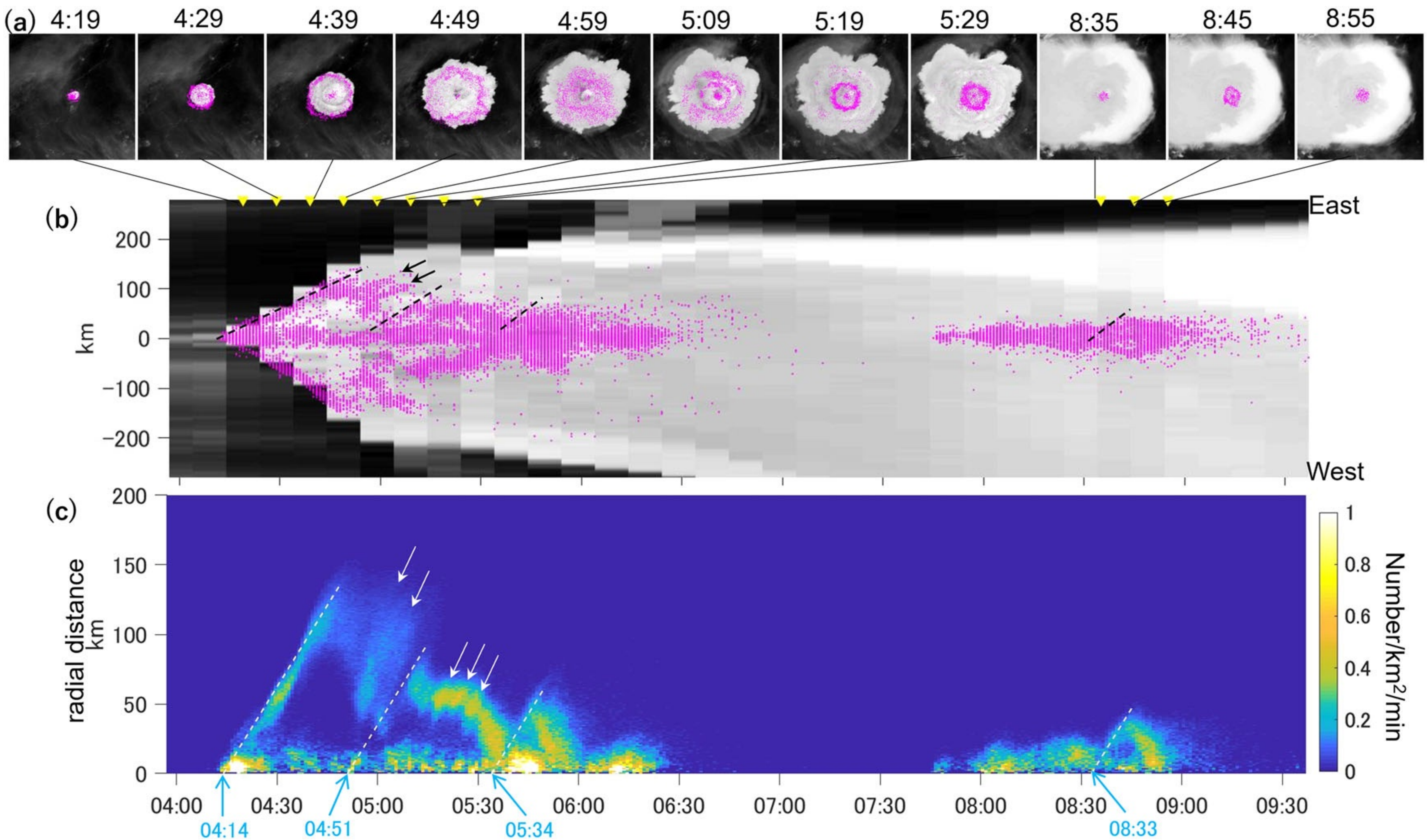


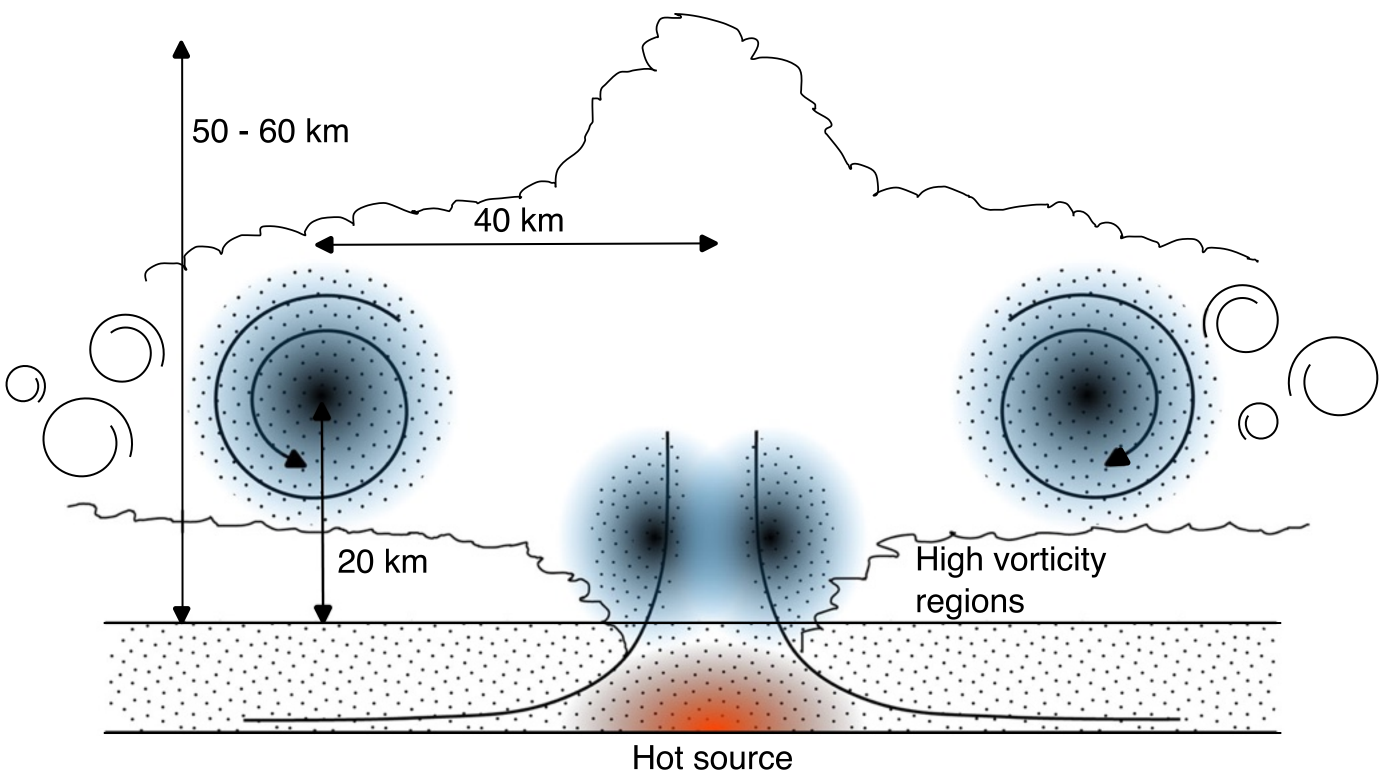
Figure 3 Illustration of the evolution of a droplet size distribution during the onset of the collision-coalescence process. Figure adapted from Berry & Reinhardt (1974) and Lamb (2001), courtesy of D. Lamb, Penn State University.

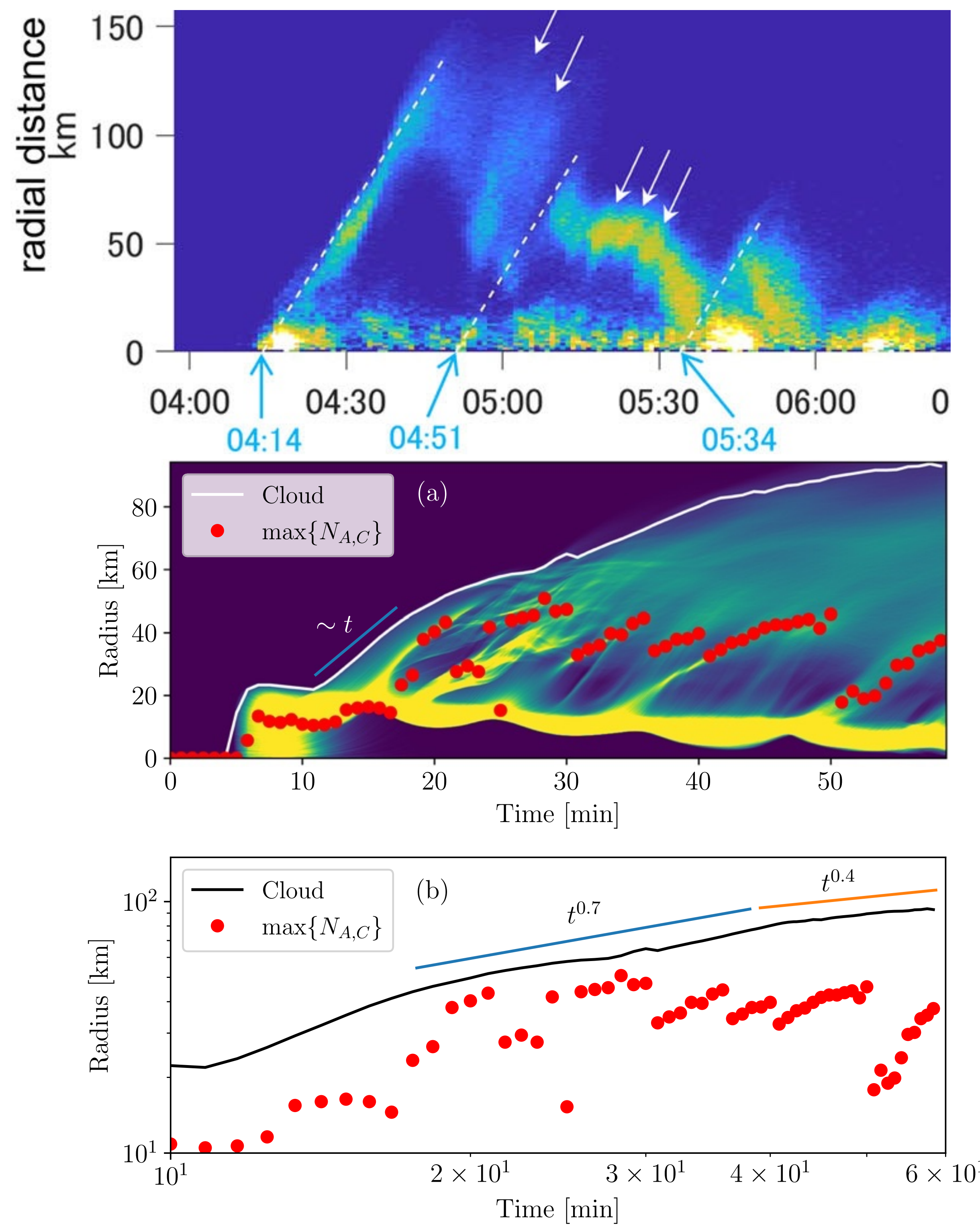
Shaw (2003)

Hunga Tonga-Hunga Ha’apai, January 15, 2022



Ring formation





Clustering:

From $\dot{\underline{v}} = -\frac{1}{\tau_p}(\underline{v} - \underline{u})$

For $St \ll 1$ (small τ_p)

$\Rightarrow \underline{v} = \underline{u} - \tau_p \underline{a} + \mathcal{O}(\tau_p^2)$

with $\underline{a} = \frac{D\underline{u}}{Dt} = \partial_t \underline{u} + \underline{u} \cdot \nabla \underline{u}$

Using that $\underline{\nabla} \cdot \underline{u} = 0$

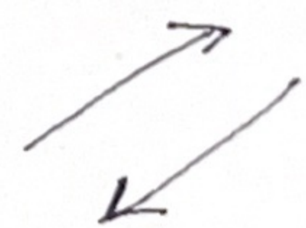
$\Rightarrow \underline{\nabla} \cdot \underline{v} = -\tau_p \, \mathrm{Tr}(\mathbb{A}^2)$

$$\Longrightarrow \quad \underline{\nabla} \cdot \underline{u} = -\tau_p \, \mathrm{Tr}\left(A^2\right)$$

with $A_{ij} = \dfrac{\partial u_i}{\partial x_j} = S_{ij} + \Omega_{ij}$

symmetric (strain-rate tensor)

antisymmetric (spin tensor)



$$\Longrightarrow \quad \underline{\nabla} \cdot \underline{u} = -\tau_p \, \mathrm{Tr}\left(S^2\right) + \tau_p \, \mathrm{Tr}\left(\Omega^2\right)$$

Regions with strain ($\mathrm{Tr}\, S^2 > \mathrm{Tr}\, \Omega^2$) have $\underline{\nabla} \cdot \underline{u} < 0 \Longrightarrow$ clusters

Regions dominated by rotation expell particles.

On the other hand, if τ_p is large,

$$\underline{v} \approx \underline{u} - \tau_p \underline{a}$$

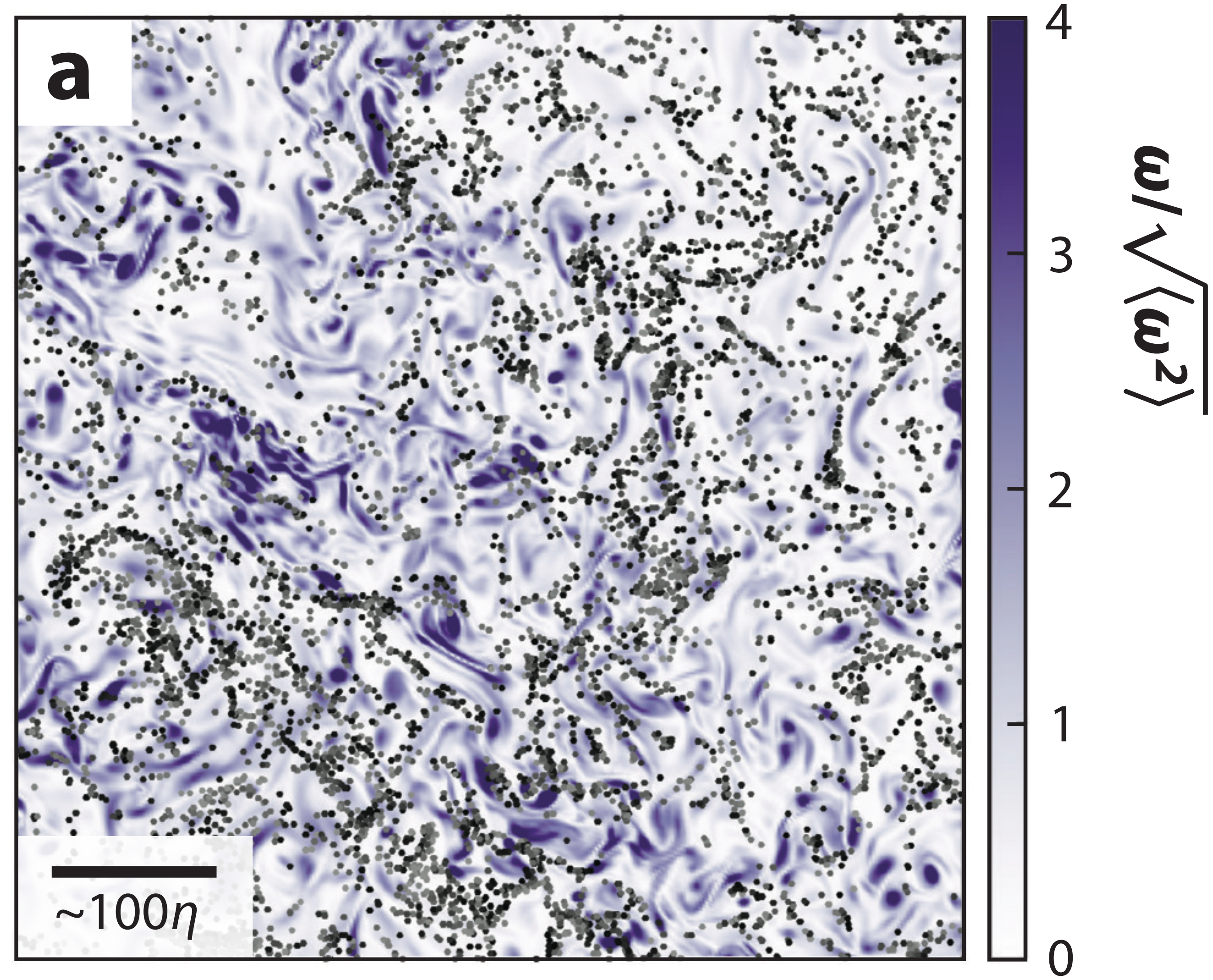
leads to particles selecting $\underline{a} \approx 0$, and these points are <u>swept</u> by the local fluid velocity $\underline{u}$.

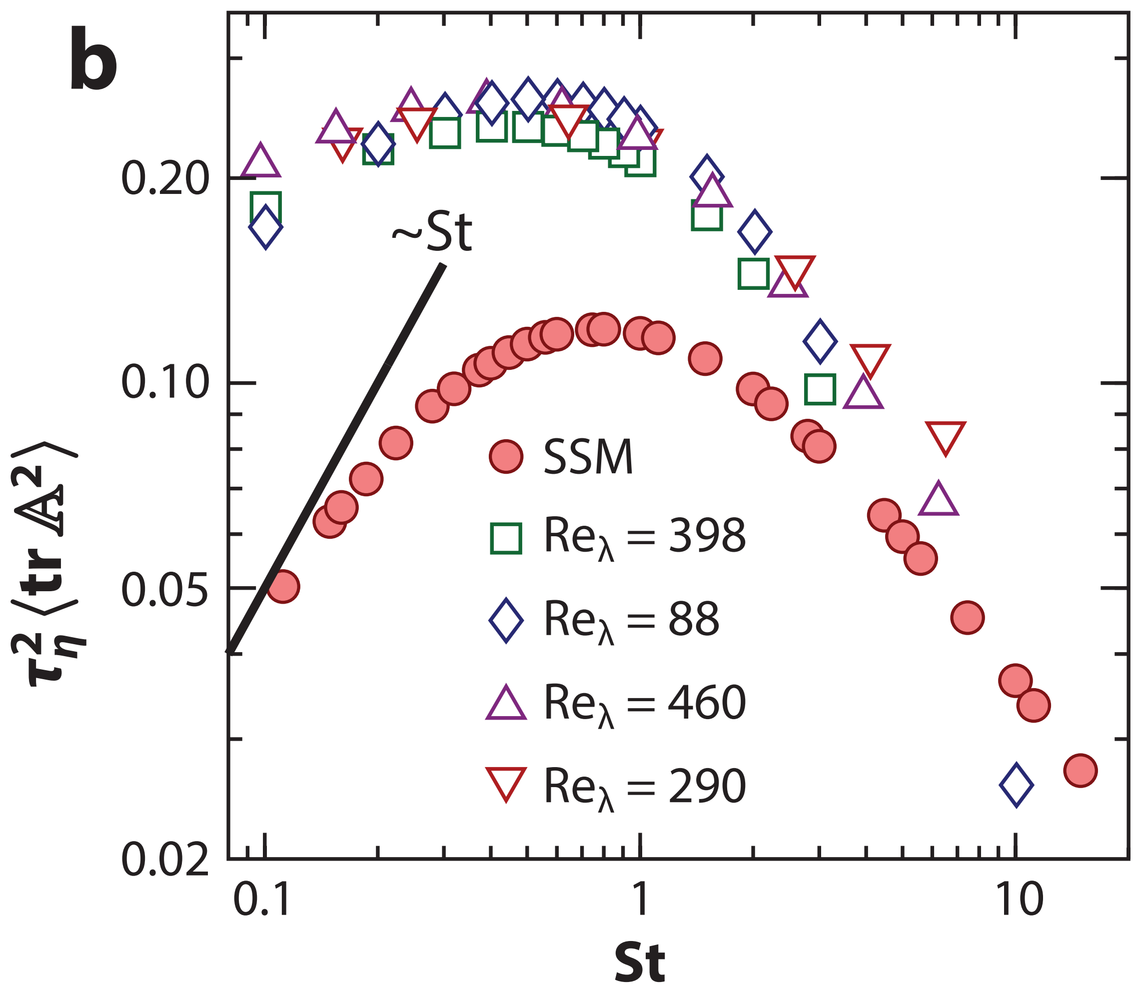
Preferential sampling

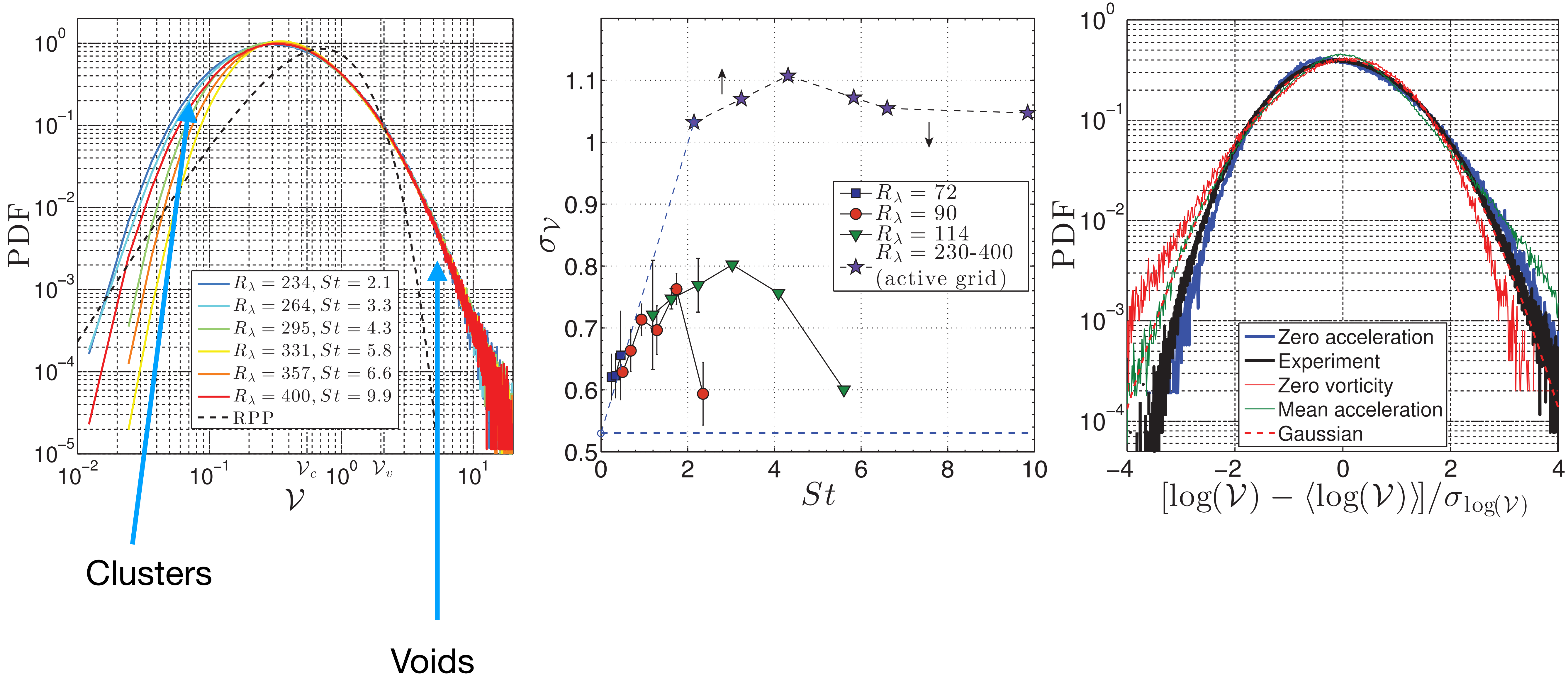
Maxey (1987): Maxey's centrifuge for St<1. Particles tend to be expelled from vortices.

Coleman and Vassilicos (2009): Sweep-stick mechanism. Particles accumulate at points of zero Lagrangian acceleration.

From Bec et al. (2014)







From Obligado et al. (2014)