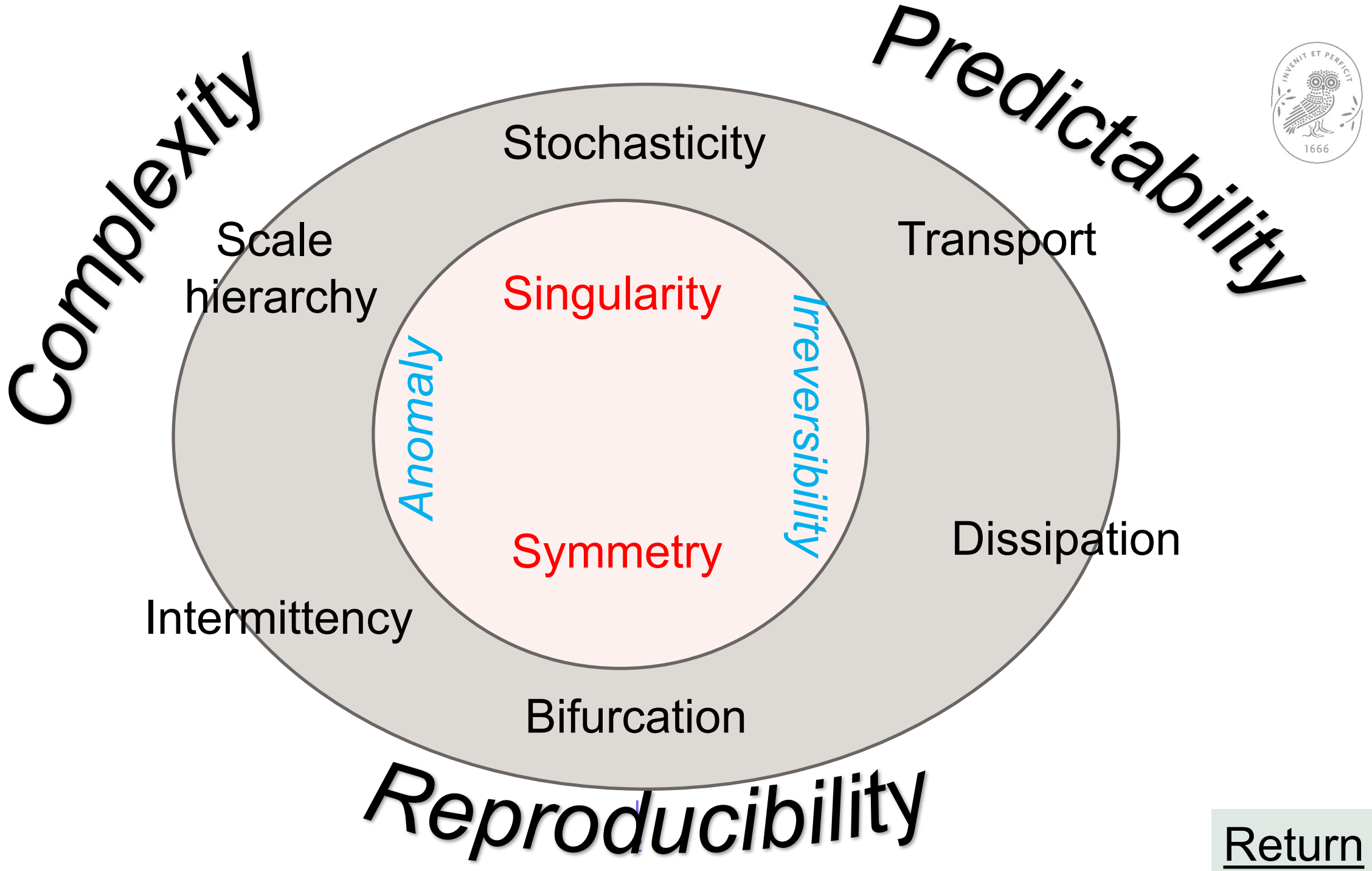
Class 5: Intermittency

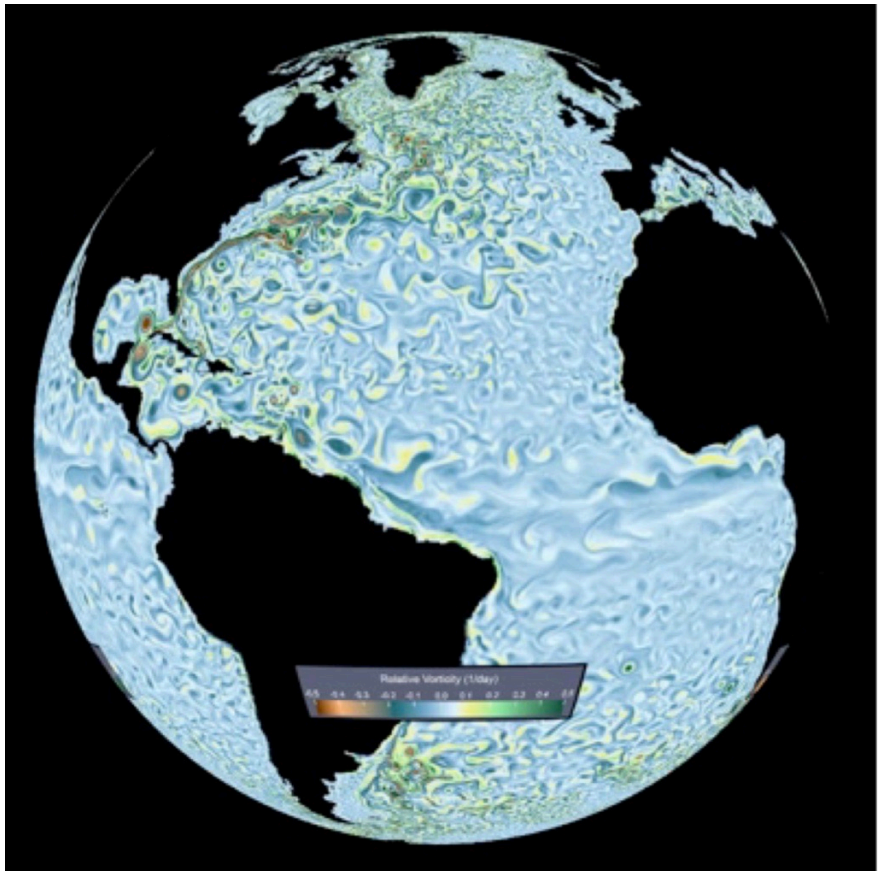
Physics of Turbulence

Rulexx: Large scale a inhomogeneous, small scale are homogeneous…

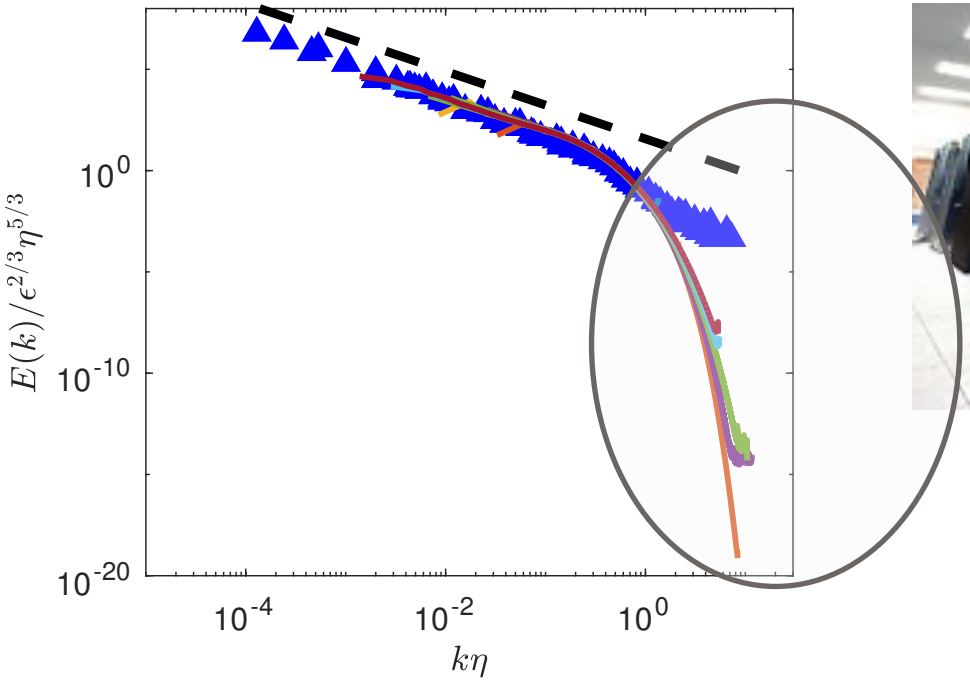

Complexity
Predictability
Reproducibility
Stochasticity
Scale hierarchy
Transport
Singularity
Anomaly
Irreversibility
Symmetry
Dissipation
Intermittency
Bifurcation

Return

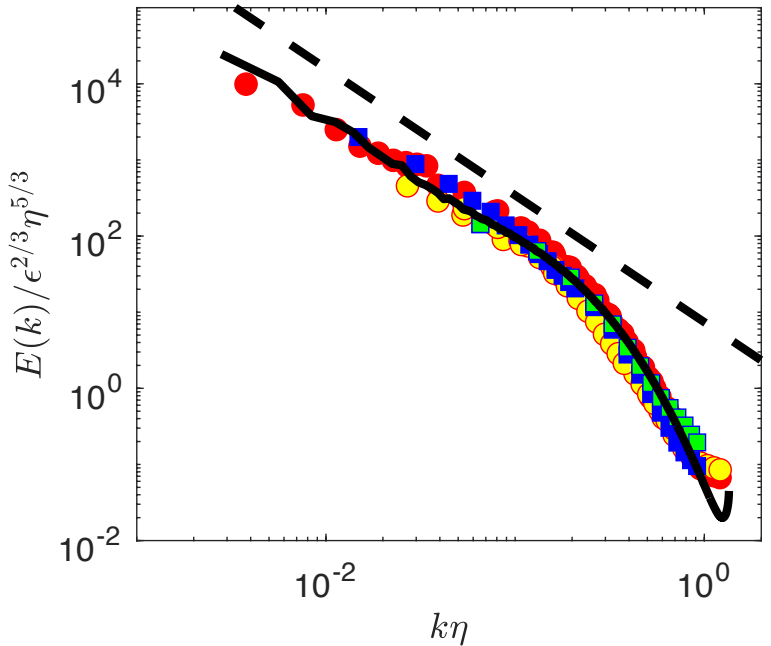
Test of K41 universality

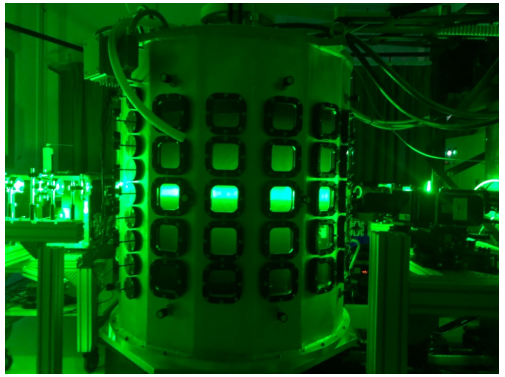


Ocean



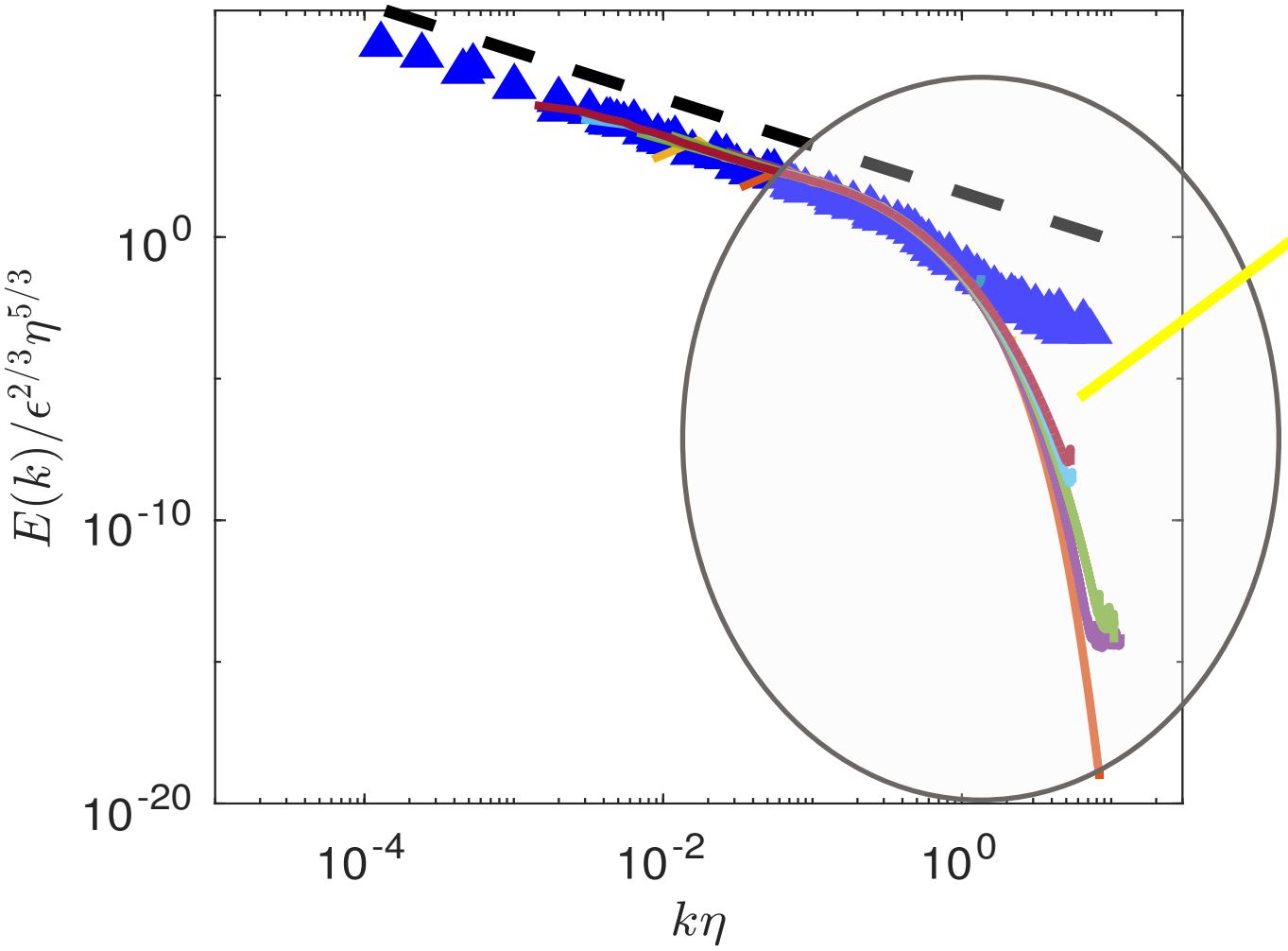
DNS



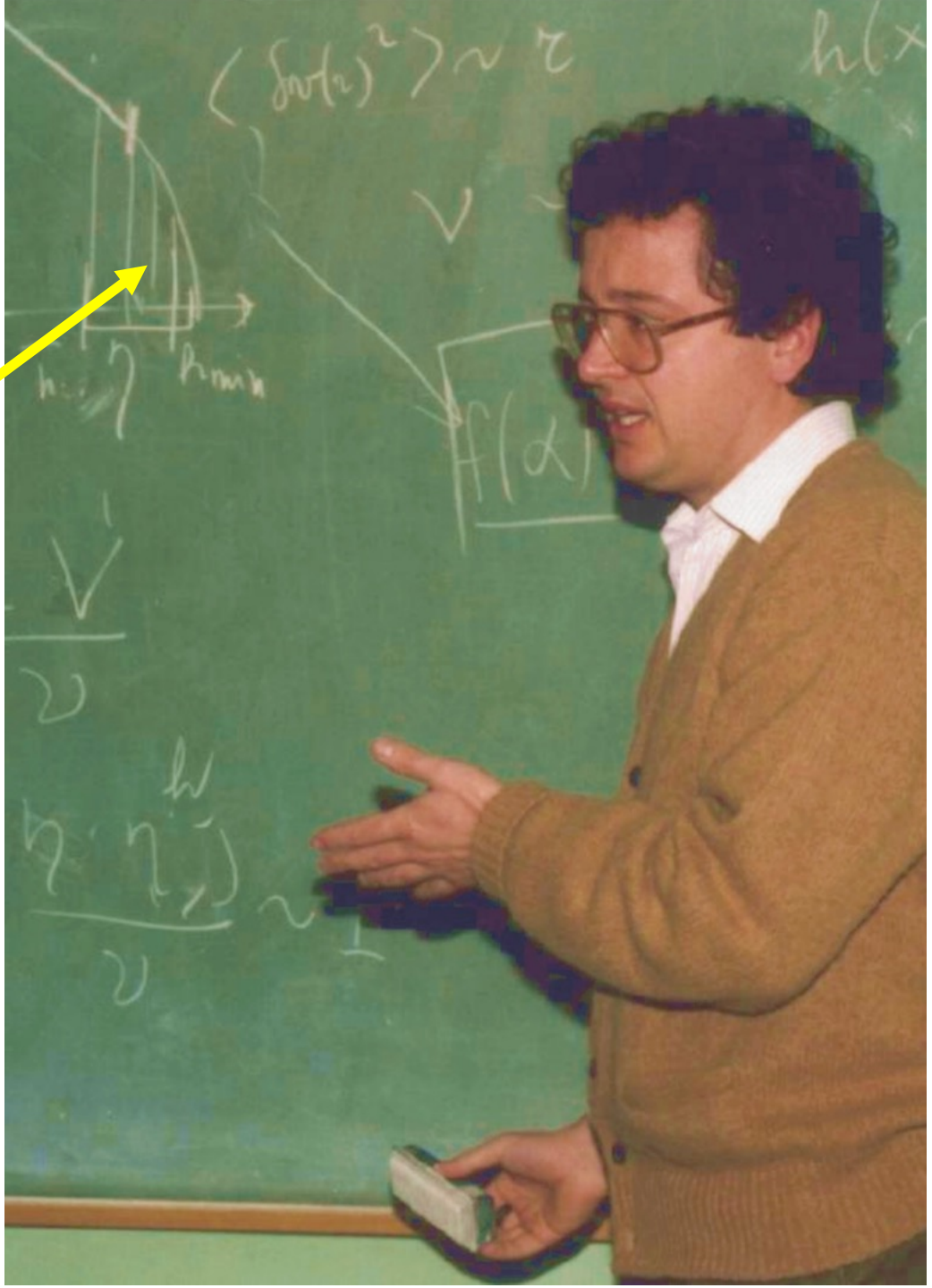


Experiments

Breaking of K41 universality



K41 appears broken in the dissipating range!



Part II: How to measure these singularities? Multifractal theory

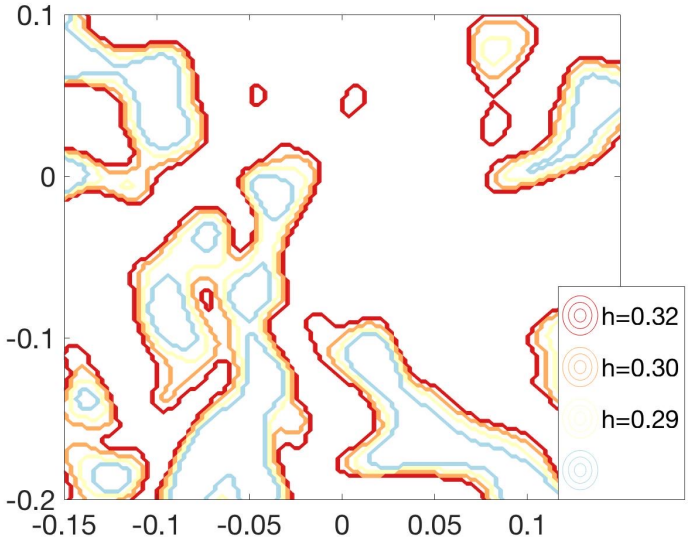
Multi-fractal theory

Heuristic interpretation of Parisi&Frisch (1985)

Replace a homogeneous fractal by a »multifractal «

$$\delta u = u(x+\ell) - u(x) = a(x)\ell^{1/3} \quad \Rightarrow \quad \delta u \sim \ell^{h(x)}$$

over a fractal set of Codimension $C(h)$



Remark If this is true, necessarily nested structure

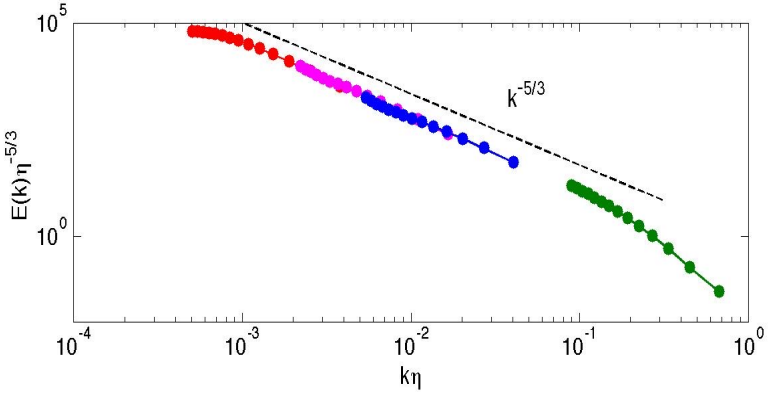
Cheskidov&Shvydkoy, 2022

Probabilistic interpretation, using large deviations

$$\text{Prob}\left[\ln(\delta u) = h\ln(\ell/L)\right] \sim_{\ell\to 0} e^{\ln(\ell/L)C(h)} = \left(\frac{\ell}{L}\right)^{C(h)},$$

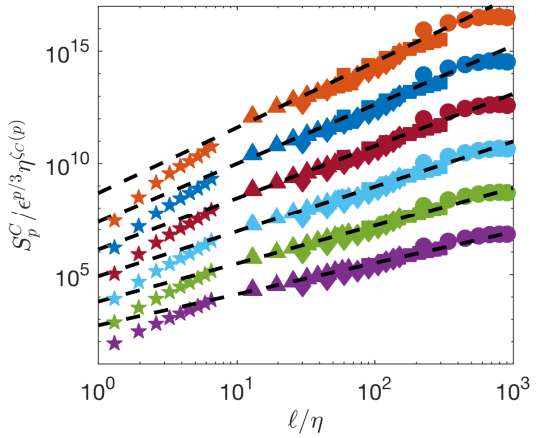
C(h): large deviation function of h

Expressing turbulent laws with C(h)



4/3 law

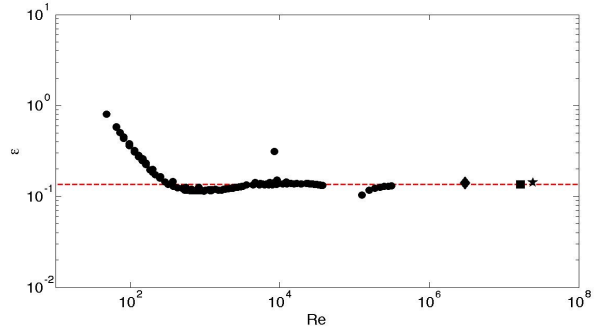
$$\langle (\delta u)^3 \rangle = \frac{4}{3}\epsilon\ell \sim \ell^{\zeta(3)}$$



Exponent Velocity Structure Functions:

$$\zeta(p) = \min_h (ph + C(h))$$

Parisi&Frisch, 1985



Mean energy dissipation:

$$\langle \epsilon \rangle \sim Re^{-\xi}$$

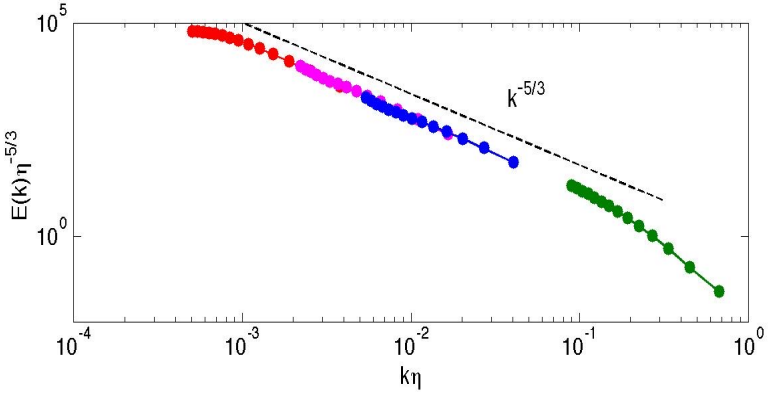
$$\xi = \min_h \left(\frac{3h - 1 + C(h)}{1 + h} \right)$$

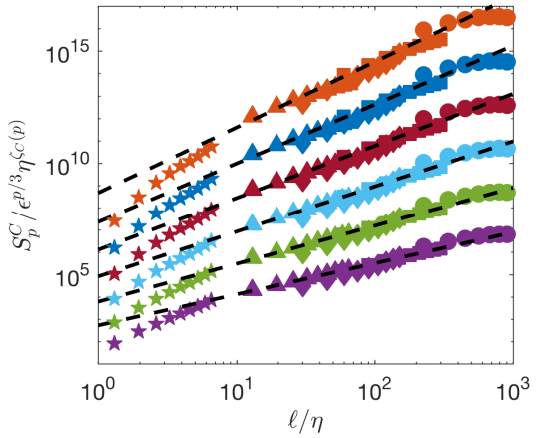
Boffetta et al, 2008

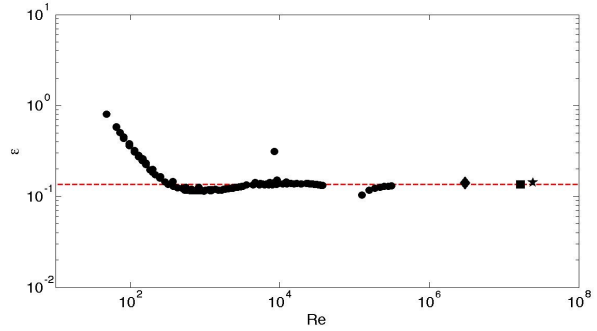
Nelkin, 1990

Benzi et al, 1991

Constraints on C(h)



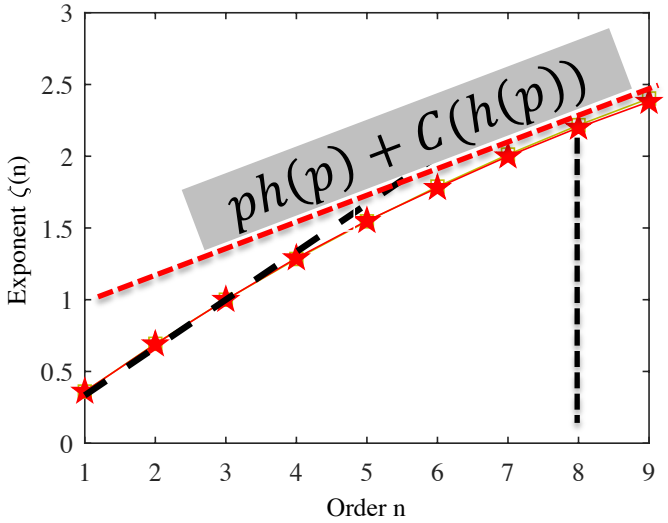




4/3 law

$$C(h) \geq 1 - 3h$$

Benzi &Biferale, 2009



Legendre Property

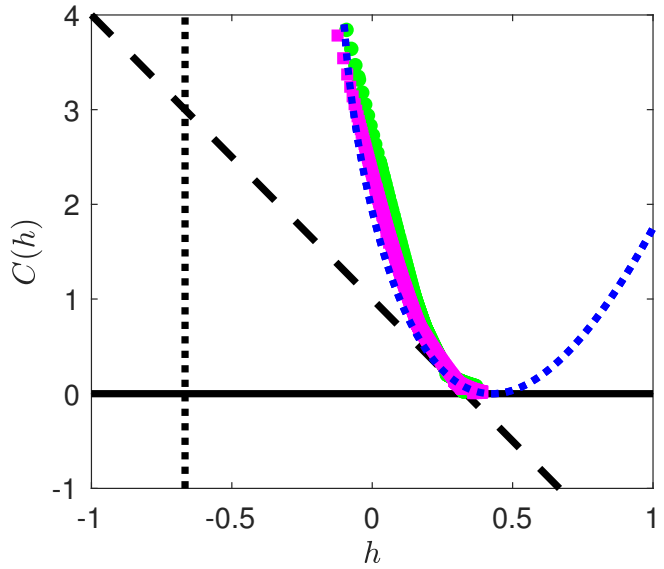
Parisi&Frisch, 1985

C is convex

Mean energy dissipation: *Boffetta et al, 2008*

$$C(h) \geq 1 - 3h$$

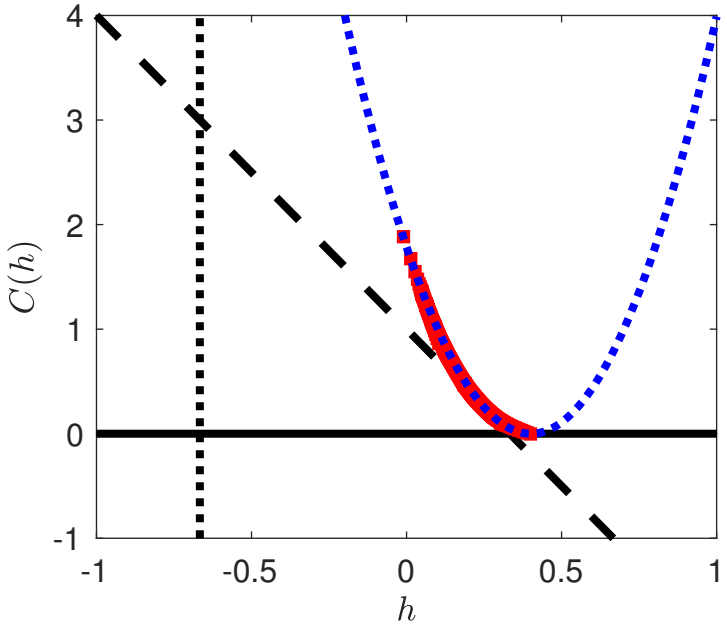
Observational constraint on C(h)



Periodic bc

Iyer, Sreenivasan&Yeung, 2020

Can be fitted by log-Poisson or log-normal



Non periodic bc

Faller et al, 2021

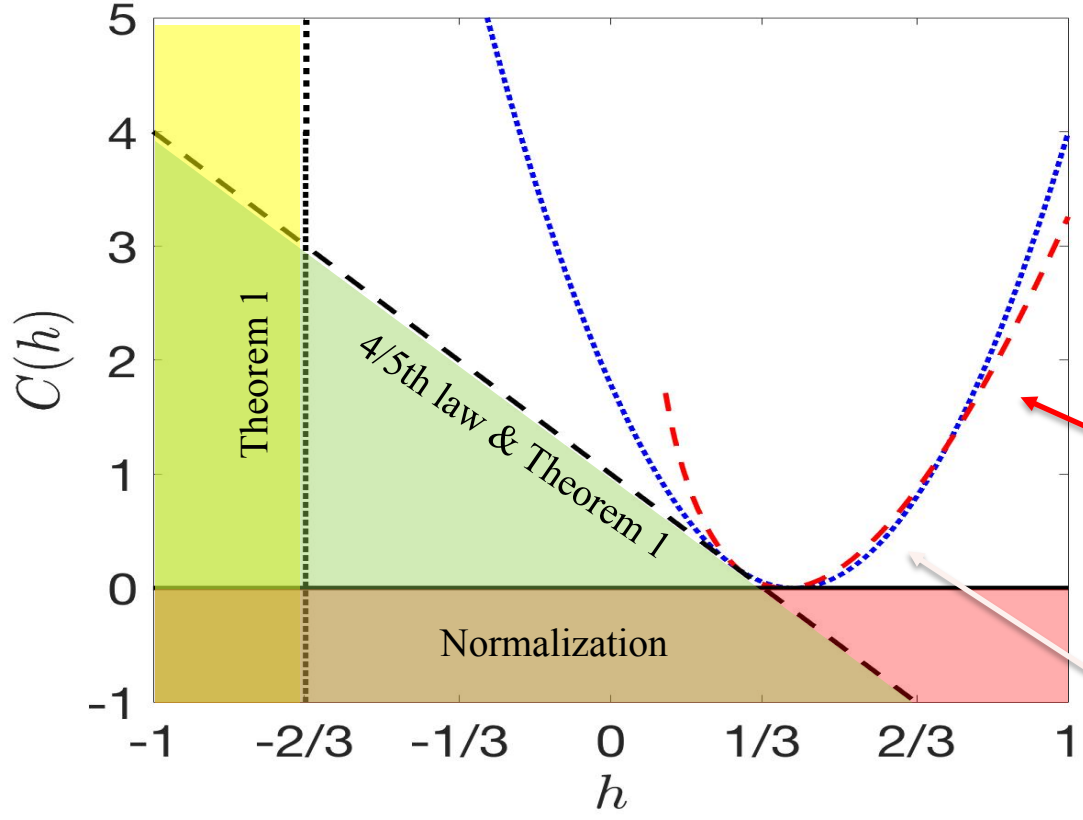
Can we say more about C(h)?

Mathematical constraints using NSE

Gibbon's theorem for weak solutions of NSE on a torus (Foias, etc)

$$\left\langle \left(\nu^{-1}\|\nabla^n u\|_{2m}\right)^{1/\alpha_{n,m}} \right\rangle_T \leq c_{n,m}\, Re^3$$

$$\alpha_{n,m} = \frac{2m}{2m(n+1)-3}$$



Impossibility to reach h=-1 for periodic boundary conditions (***Dubrulle&Gibbon, 2022***)

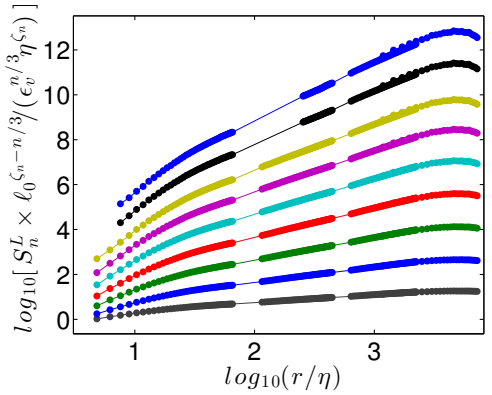
Log-Poisson

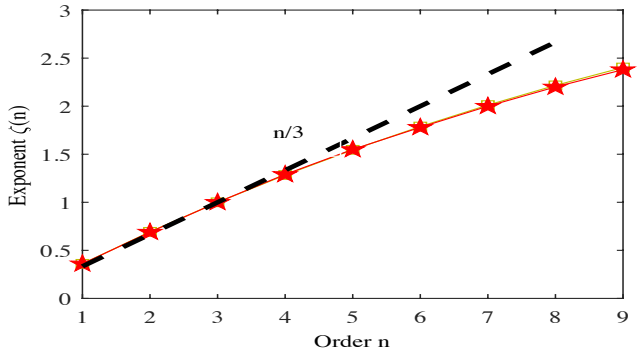
$$C(h) = \frac{h-h_0}{\ln\beta} + h_-\left[1-(h-h_0)/h_-\right]\ln\left[1-(h-h_0)/h_-\right];$$

Log-normal

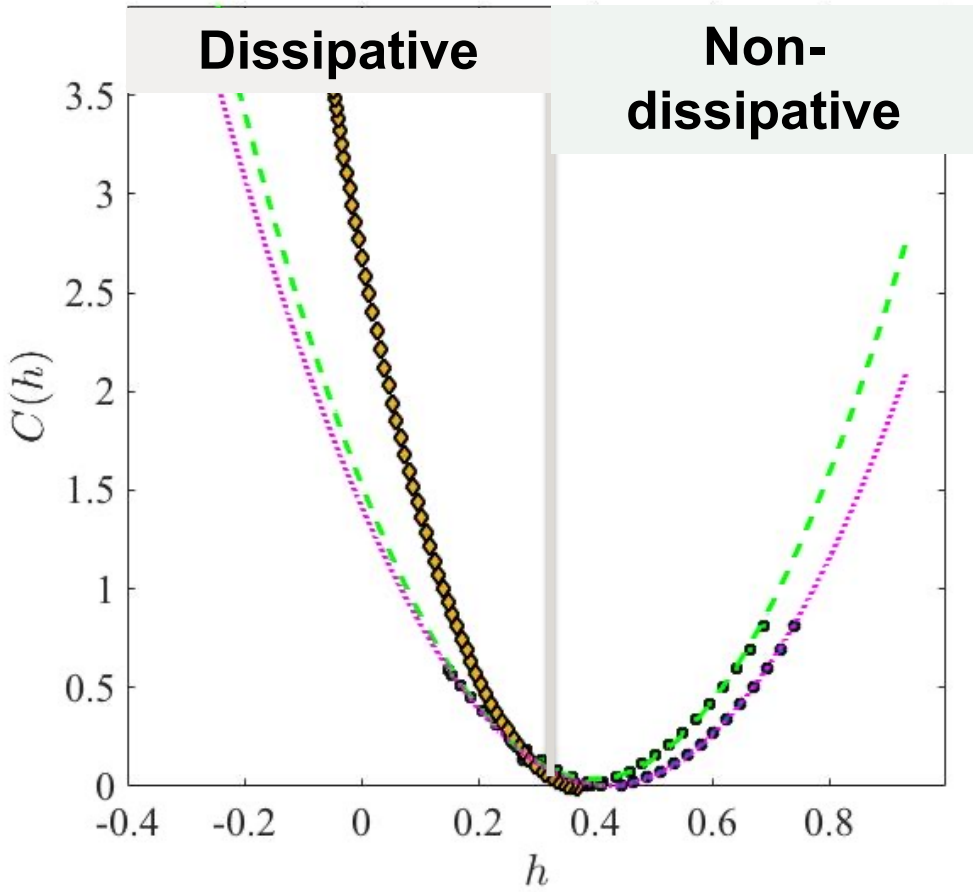
$$C(h) = \frac{(h-h_0)^2}{2\,(h_0-\zeta(1))}$$

Dissipative vs non-dissipative (Onsager)



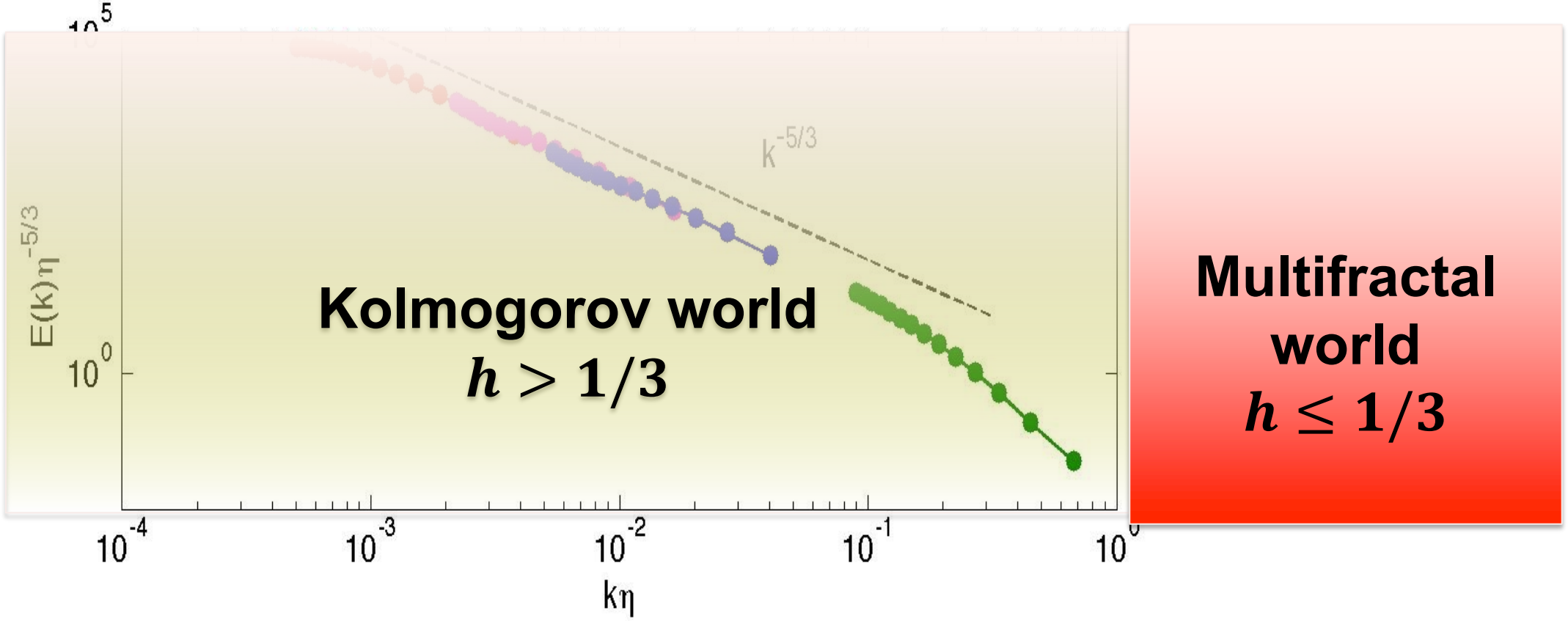


Order n>0 Order n<0



C(h): large deviation function of h

The new world beyond Kolmogorov scale



To observe these structures necessity to go at very large resolution!!!

-> **Challenge for DNS and experiments!**