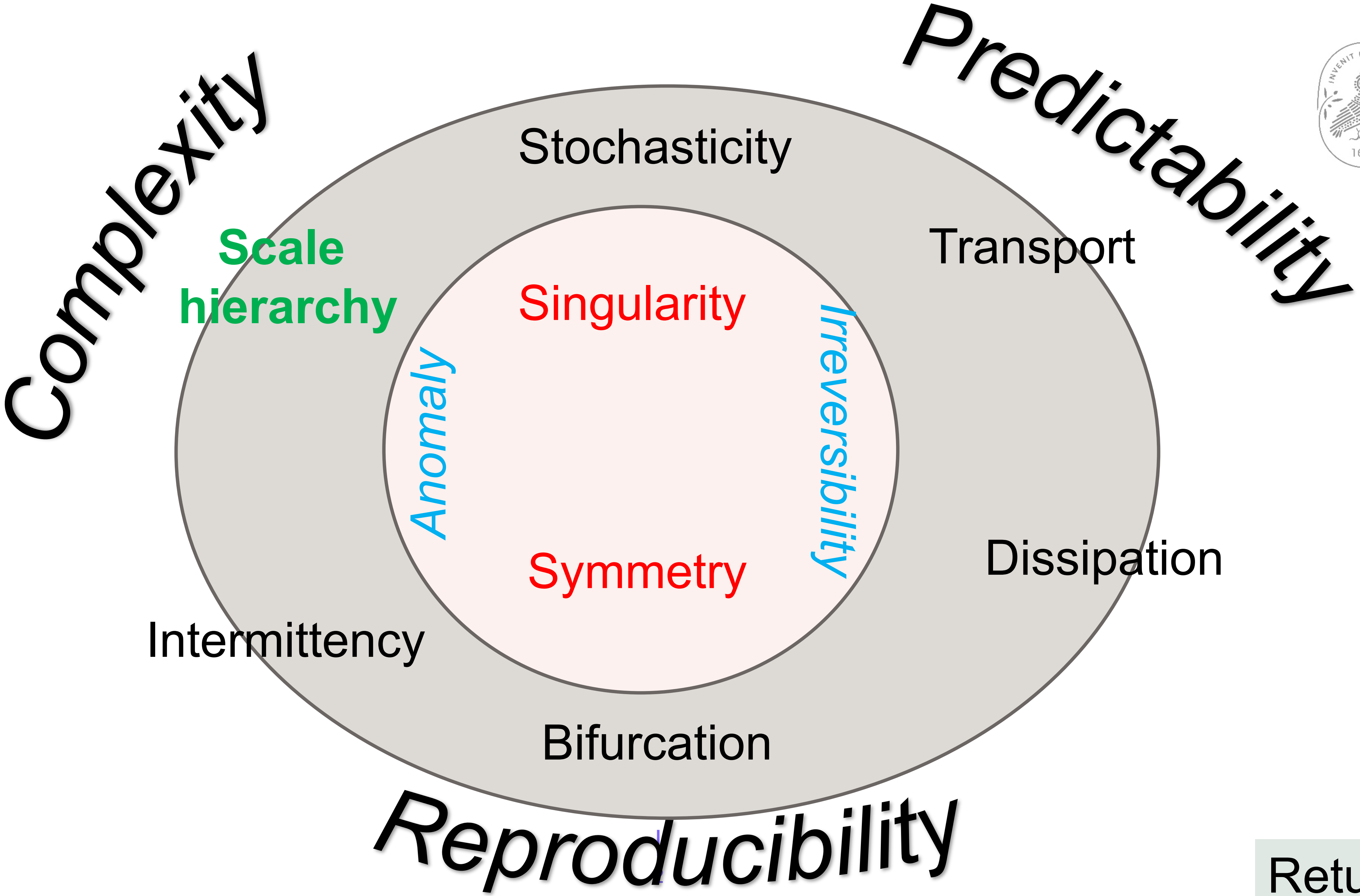
Class 3: the Scale hierarchy

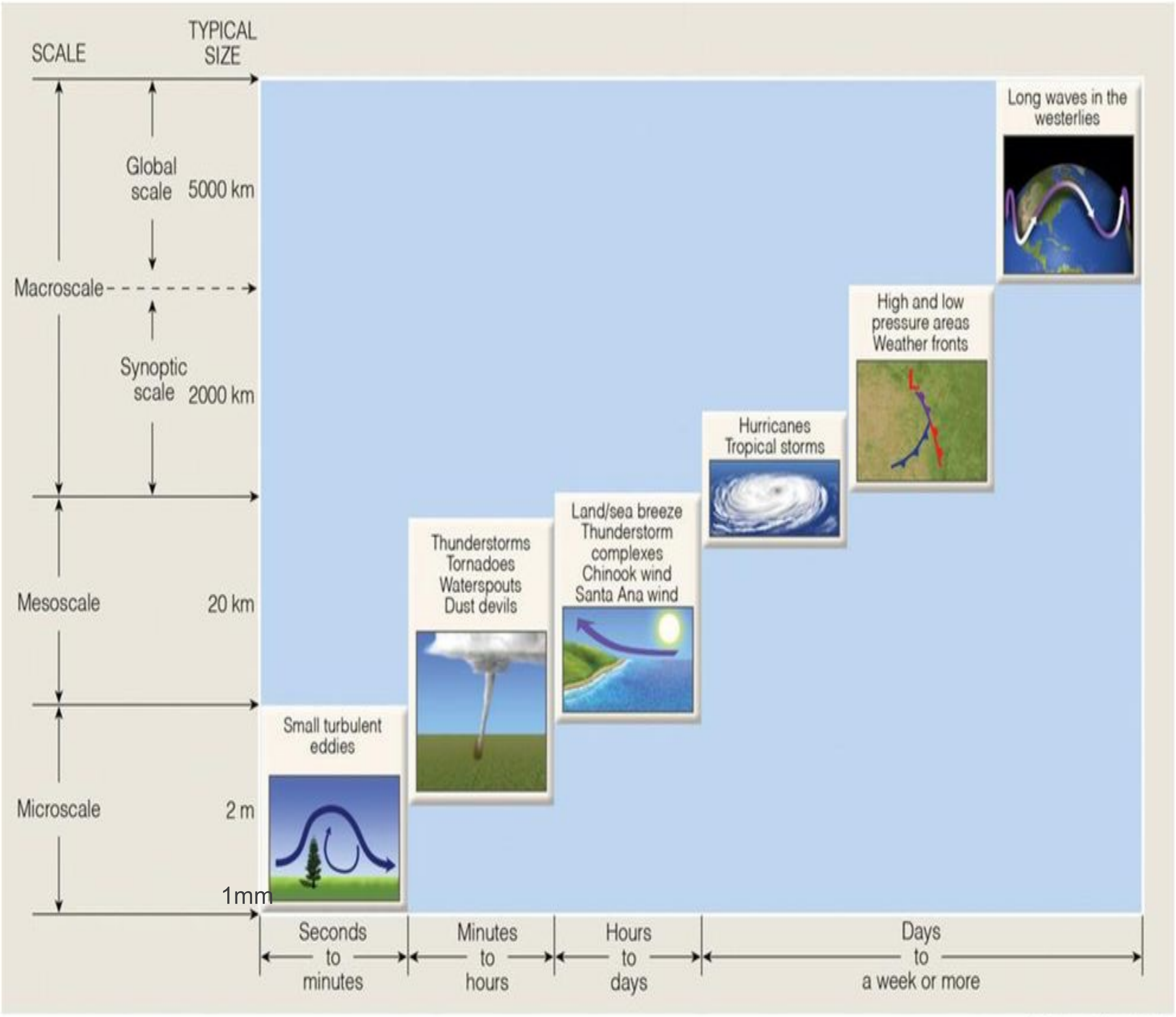
Physics of Turbulence

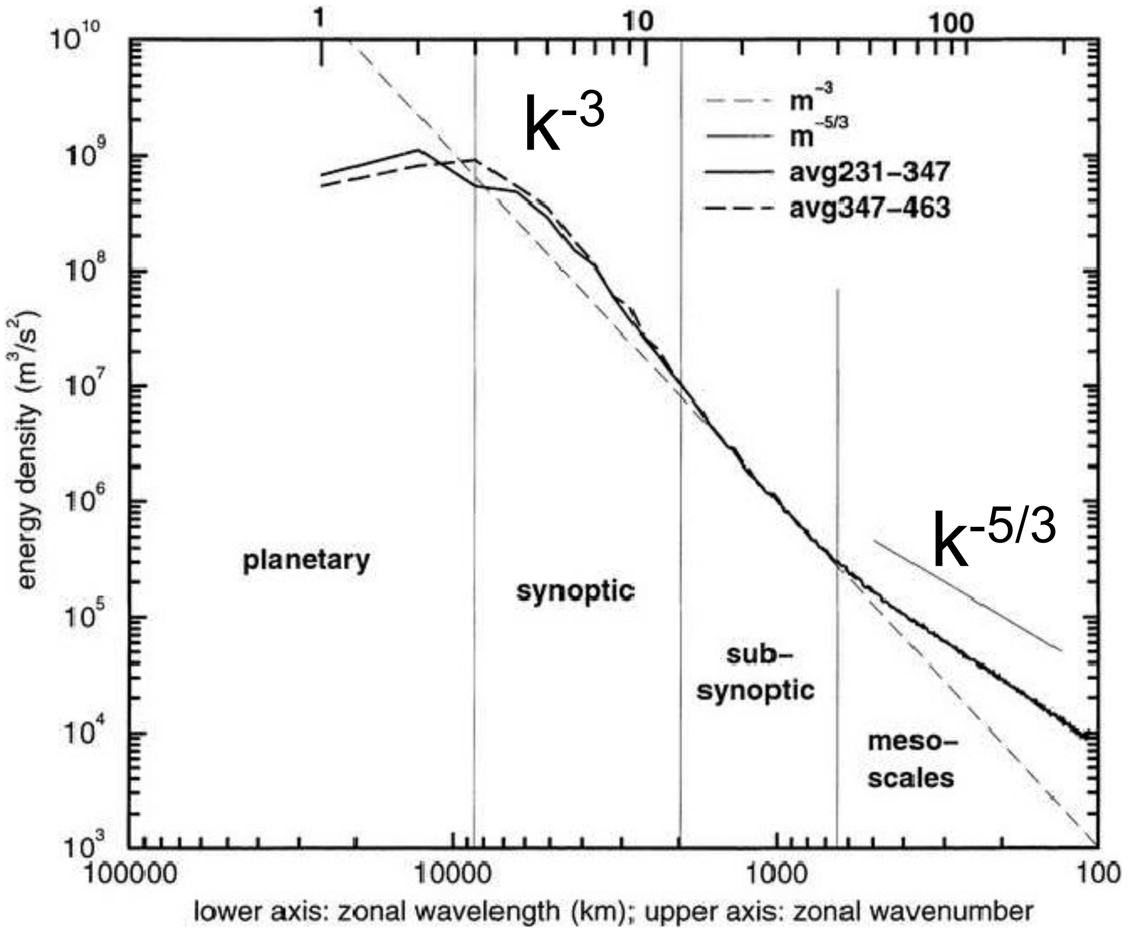

Complexity
Predictability
Reproducibility
Stochasticity
Scale hierarchy
Transport
Singularity
Anomaly
Irreversibility
Symmetry
Dissipation
Intermittency
Bifurcation

Return

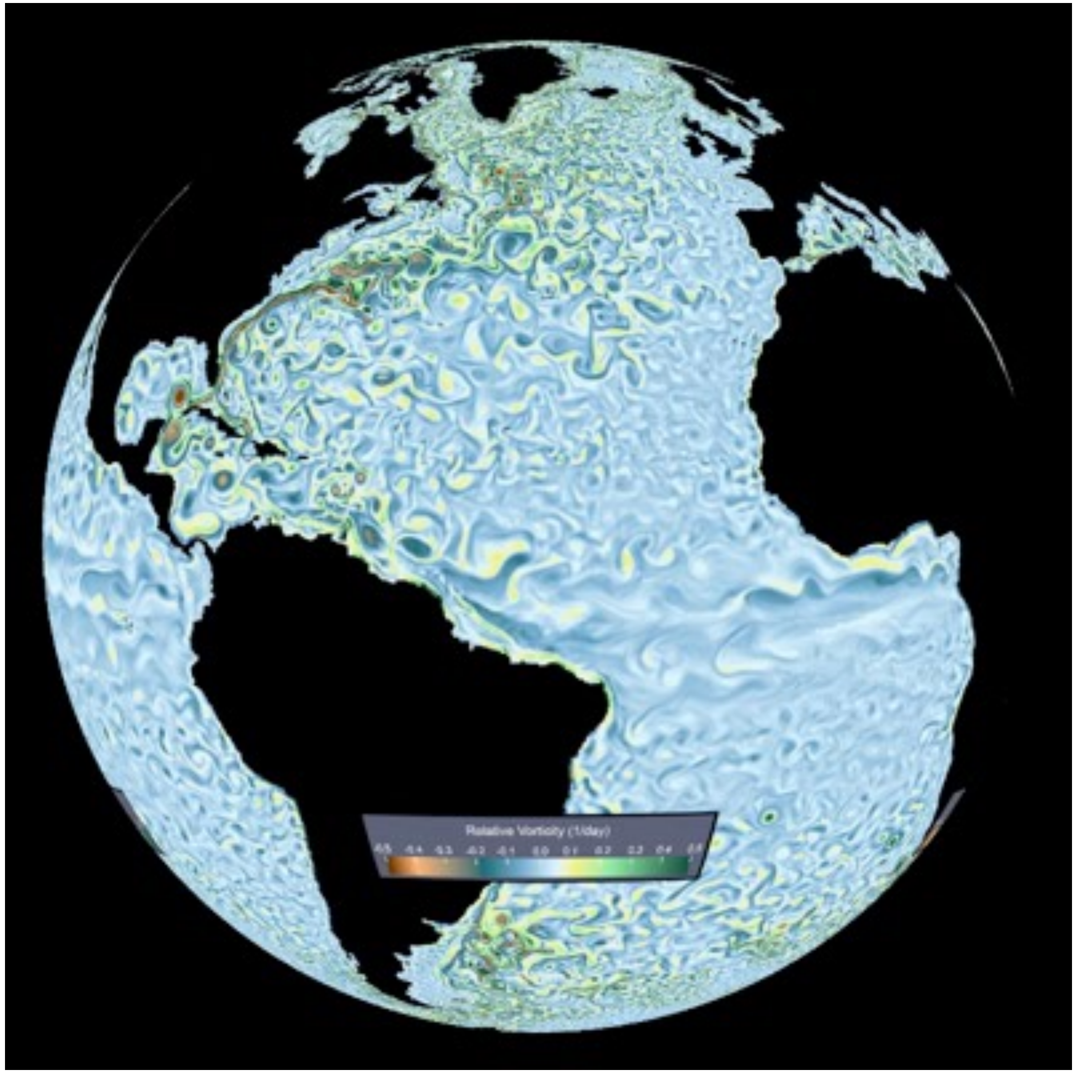
Scale hierachy

Nastrom-Gage spectrum





Scale hierarchy



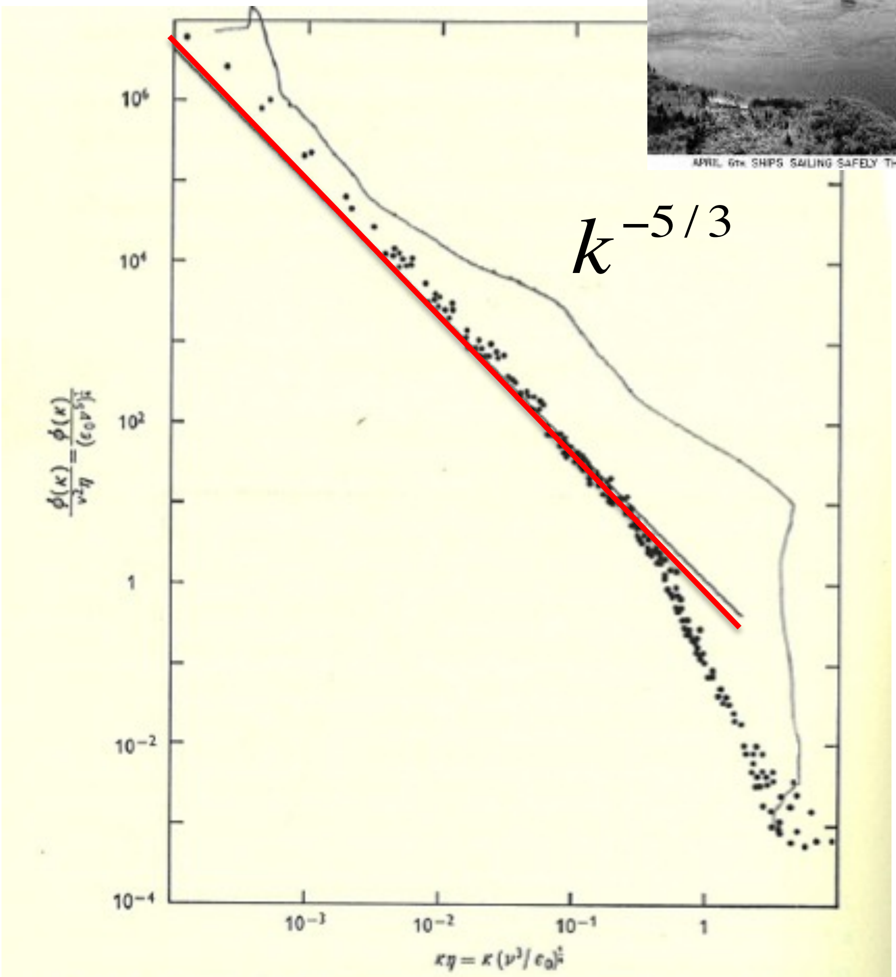


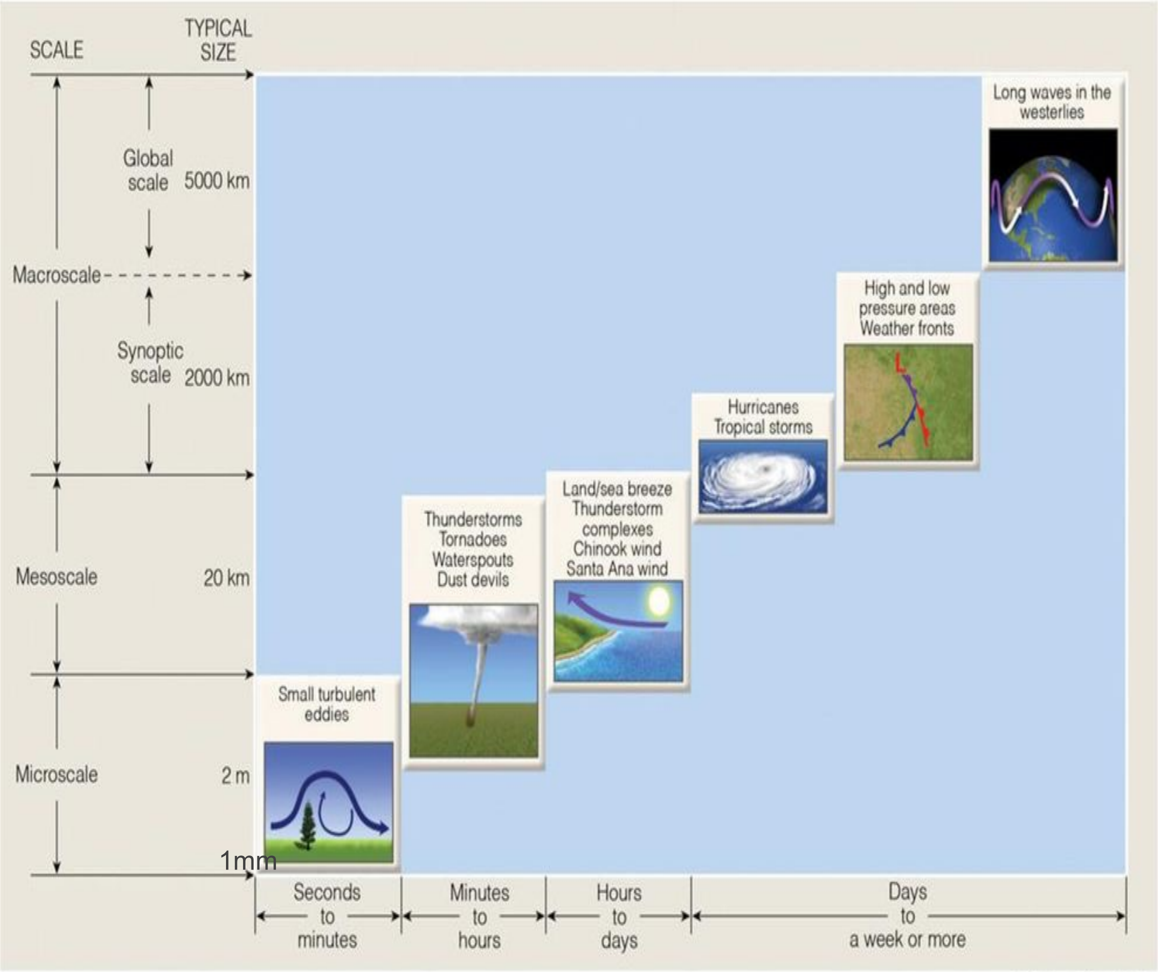
Fig. 6.2. The turbulence spectra, measured by Grant, Stewart and Moilliet (1962) and scaled according to the Kolmogorov parameters. The viscous dissipation rate ϵ_0 varied over a range of values of the order 100. The straight line represents variation as $\kappa^{-\frac{5}{3}}$. The top few points are believed to be rather high on account of the low frequency heaving motions of the ship.

APRIL 6th SHIPS SAILING SAFELY THROUGH SEYMOUR NARROWS.

Vortices are organized in a hierarchical way
They are regularized by viscosity

Can we explain the difference?

Weather equations



$$\left(\frac{\partial}{\partial t} + \boldsymbol{v}_{||} \cdot \nabla_{||} + w\frac{\partial}{\partial z}\right)\boldsymbol{v}_{||} + \varepsilon(2\boldsymbol{\Omega} \times \boldsymbol{v})_{||} + \frac{1}{\varepsilon^3 \rho}\nabla_{||} p = \boldsymbol{Q}_{v_{||}},$$

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v}_{||} \cdot \nabla_{||} + w\frac{\partial}{\partial z}\right) w + \varepsilon(2\boldsymbol{\Omega} \times \boldsymbol{v})_{\perp} + \frac{1}{\varepsilon^3 \rho}\frac{\partial p}{\partial z} = Q_w - \frac{1}{\varepsilon^3},$$

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v}_{||} \cdot \nabla_{||} + w\frac{\partial}{\partial z}\right)\rho + \rho\nabla \cdot \boldsymbol{v} = 0,$$

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v}_{||} \cdot \nabla_{||} + w\frac{\partial}{\partial z}\right)\Theta = Q_{\Theta}.$$

Present weather modelling

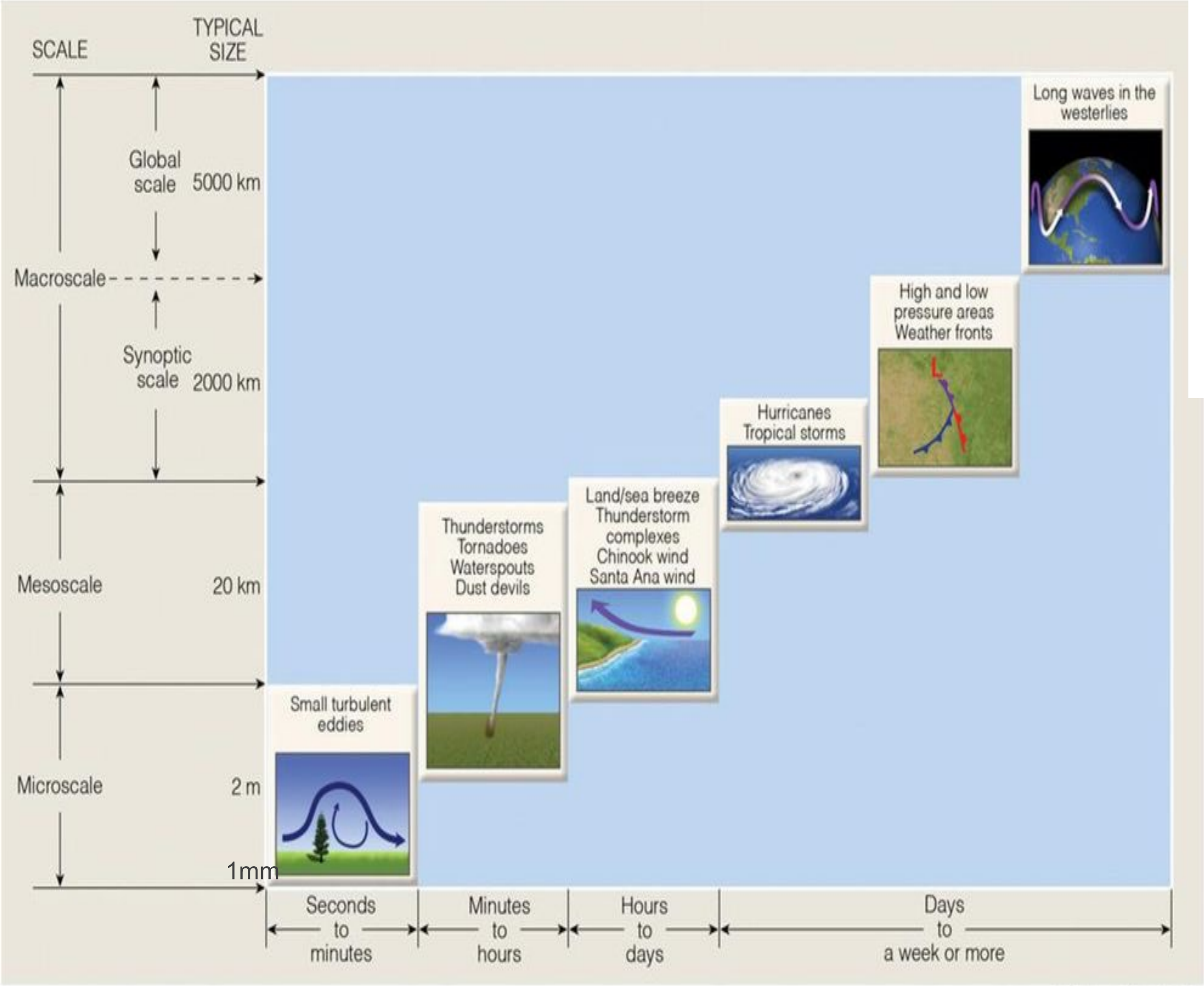


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Table 1 Universal characteristics of atmospheric motions

Earth's radius	$a \sim 6 \times 10^6$ m
Earth's rotation rate	$\Omega \sim 10^{-4}$ s^{-1}
Acceleration of gravity	$g \sim 9.81$ ms^{-2}
Sea-level pressure	$p_{ref} \sim 10^5$ kgm^{-1} s^{-2}
H_2O freezing temperature	$T_{ref} \sim 273$ K
Equator-pole potential temperature difference Tropospheric vertical potential temperature difference	$\Delta\Theta \sim 40$ K
Dry gas constant	$R = 287$ m^2 s^{-2} K^{-1}
Dry isentropic exponent	$\gamma = 1.4$

7 equations, 3 dimensions
4 independent non-dimensional parameters

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Scale dependent equations

Table 2 Auxiliary quantities of interest derived from those in Table 1

Sea-level air density	$\rho_{\mathrm{ref}} = p_{\mathrm{ref}}/(RT_{\mathrm{ref}}) \sim 1.25\ \mathrm{kgm}^{-3}$
Density scale height	$h_{\mathrm{sc}} = \gamma p_{\mathrm{ref}}/(g\rho_{\mathrm{ref}}) \sim 11\ \mathrm{km}$
Sound speed	$c_{\mathrm{ref}} = \sqrt{\gamma p_{\mathrm{ref}}/\rho_{\mathrm{ref}}} \sim 330\ \mathrm{ms}^{-1}$
Internal wave speed	$c_{\mathrm{int}} = \sqrt{g h_{\mathrm{sc}} \frac{\Delta\Theta}{T_{\mathrm{ref}}}} \sim 110\ \mathrm{ms}^{-1}$
Thermal wind velocity	$u_{\mathrm{ref}} = \frac{2}{\pi}\frac{g h_{\mathrm{sc}}}{\Omega a}\frac{\Delta\Theta}{T_{\mathrm{ref}}} \sim 12\ \mathrm{ms}^{-1}$

$$u_{ref}/c_{int} \sim \epsilon$$

$$u_{ref}/c_{ref} \sim \epsilon^{3/2}$$

$$c_{int}/c_{ref} \sim \epsilon^{1/2}$$

$$L_{\mathrm{meso}} = \frac{h_{\mathrm{sc}}}{\varepsilon}, \quad L_{\mathrm{Ro}} = \frac{h_{\mathrm{sc}}}{\varepsilon^2}, \quad L_{\mathrm{Ob}} = \frac{h_{\mathrm{sc}}}{\varepsilon^{\frac{5}{2}}}, \quad L_p = \frac{h_{\mathrm{sc}}}{\varepsilon^3}.$$

Table 3 Hierarchy of physically distinguished scales in the atmosphere

Planetary scale	$L_{\mathrm{p}} = \frac{\pi}{2} a \sim 10000\ \mathrm{km}$
Obukhov radius	$L_{\mathrm{Ob}} = \frac{c_{\mathrm{ref}}}{\Omega} \sim 3300\ \mathrm{km}$
Synoptic scale	$L_{\mathrm{Ro}} = \frac{c_{\mathrm{int}}}{\Omega} \sim 1100\ \mathrm{km}$
Meso-β scale	$L_{\mathrm{meso}} = \frac{u_{\mathrm{ref}}}{\Omega} \sim 150\ \mathrm{km}$
Meso-γ scale	$h_{\mathrm{sc}} = \frac{\gamma p_{\mathrm{ref}}}{g\rho_{\mathrm{ref}}} \sim 11\ \mathrm{km}$

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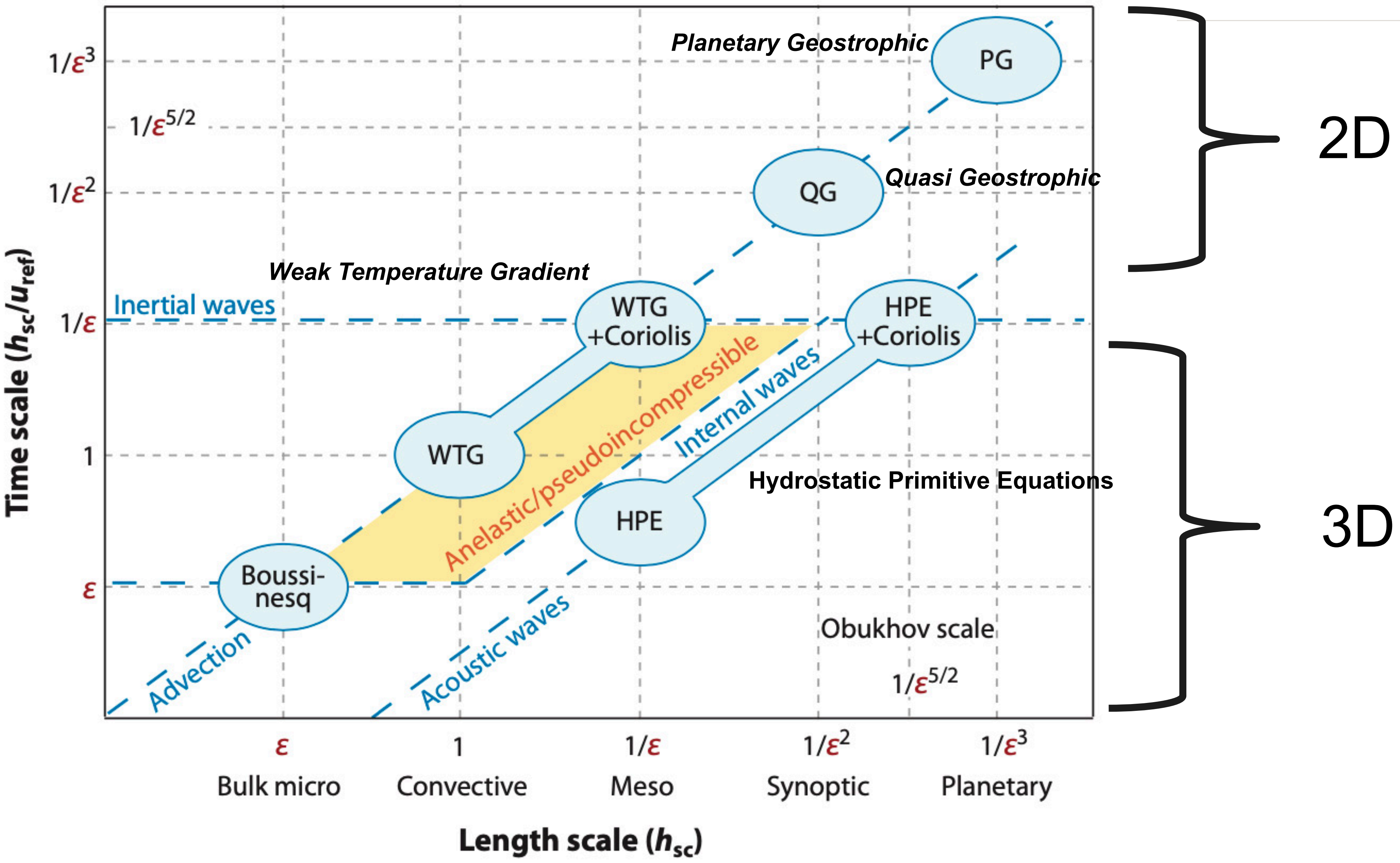
Scale dependent equations



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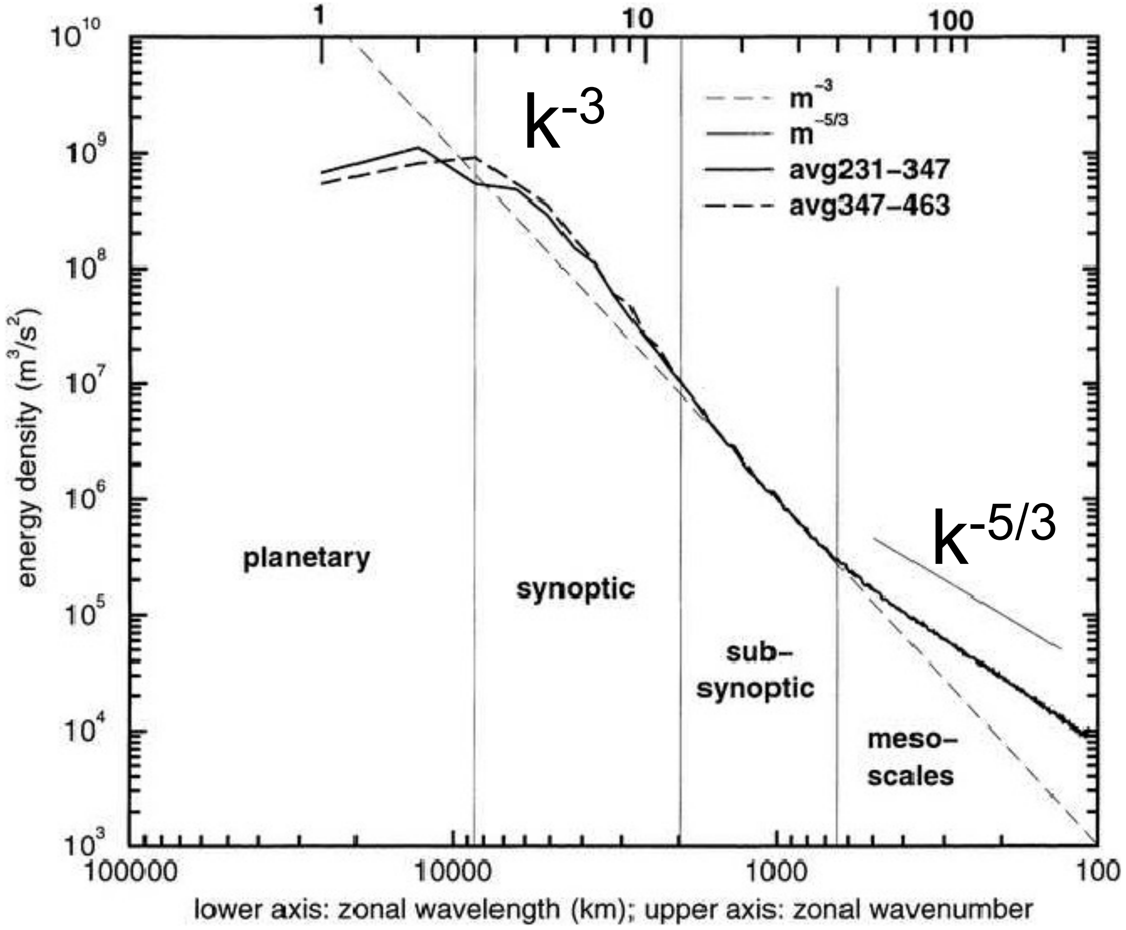
2D vs 3D

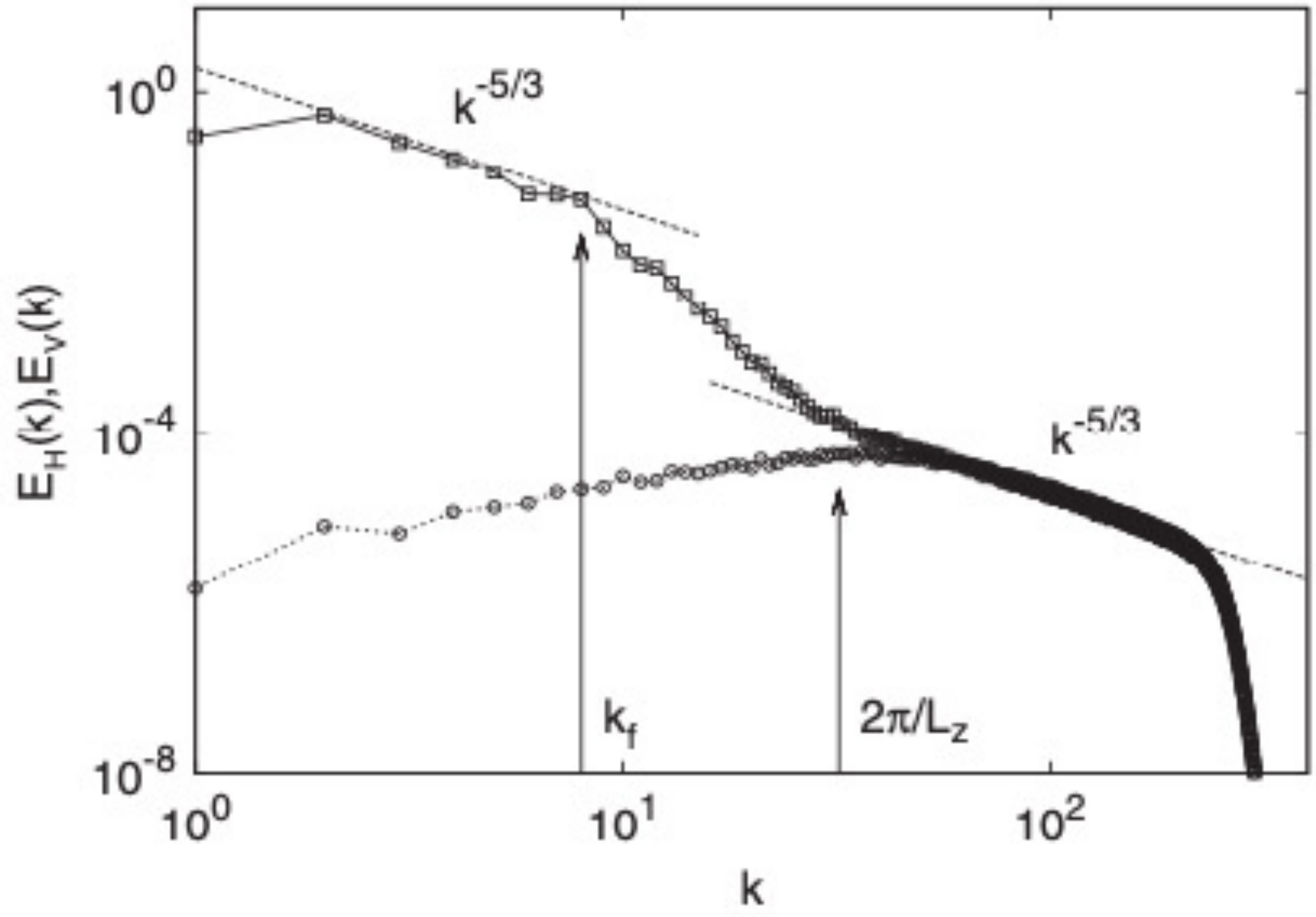


1
10
100
10^{10}
10^{9}
10^{8}
10^{7}
10^{6}
10^{5}
10^{4}
10^{3}
energy density (m^3/s^2)
k^{-3}
$k^{-5/3}$
m^{-3}
$m^{-5/3}$
avg231–347
avg347–463
planetary
synoptic
sub-
synoptic
meso-
scales
100000
10000
1000
100
lower axis: zonal wavelength (km); upper axis: zonal wavenumber

2D

3D

2D vs 3D



2D

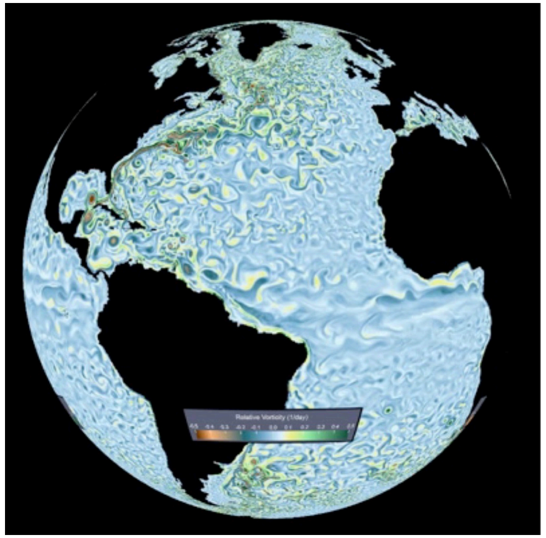
3D

Navier Stokes simulation
in an anisotropic domain

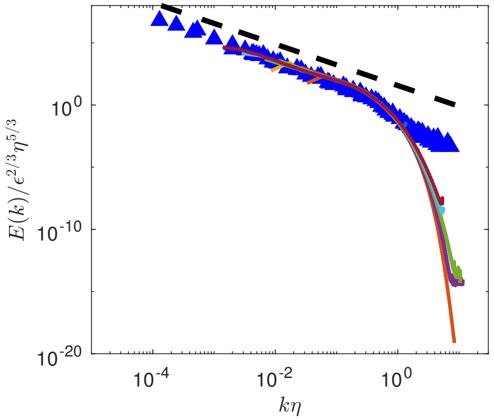
Celani et al, PRL, 2010

3D energy spectrum and Kolmogorov theory

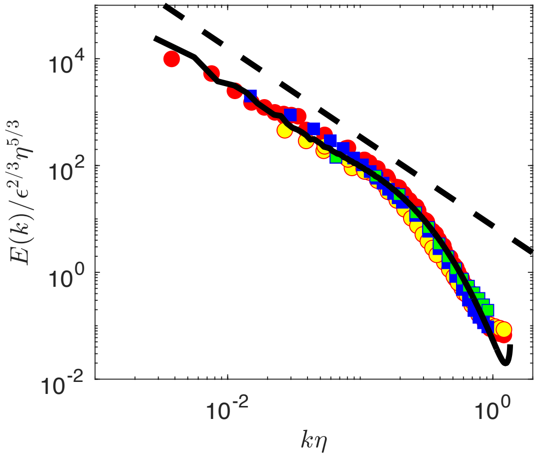
K41 universality

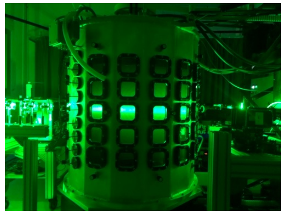


Ocean



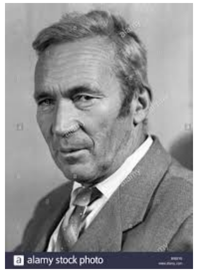
DNS





Experiments

1941:Kolmogorov Theory



NSE+homogeneity

$$E =< (\delta u)^2 >$$

$$\frac{1}{4}\partial_t E + \epsilon = -\frac{1}{4}\nabla_\ell < (\delta u)^3 > + \frac{1}{2}\nu \Delta_\ell E + P_{inj}$$

$$\delta u_\ell = u(x+\ell) - u(x)$$

Karman Howarth equation

Kolmogorov Theory (2)

KH equation + **self-similarity**+**stationarity**

$$\frac{1}{4}\cdots + \epsilon = -\frac{1}{4}\nabla_\ell < (\delta u)^3 > + \cdots{}_\ell E + \cdots$$

$$\left\langle \left(\delta u_\ell\right)^3 \right\rangle \propto -\frac{4}{3}\varepsilon \ell$$

$$\left\langle \left(\delta u_\ell\right)^2 \right\rangle \propto \left(\varepsilon \ell\right)^{2/3}$$

$$E(k) = C\,\varepsilon^{2/3} k^{-5/3}$$