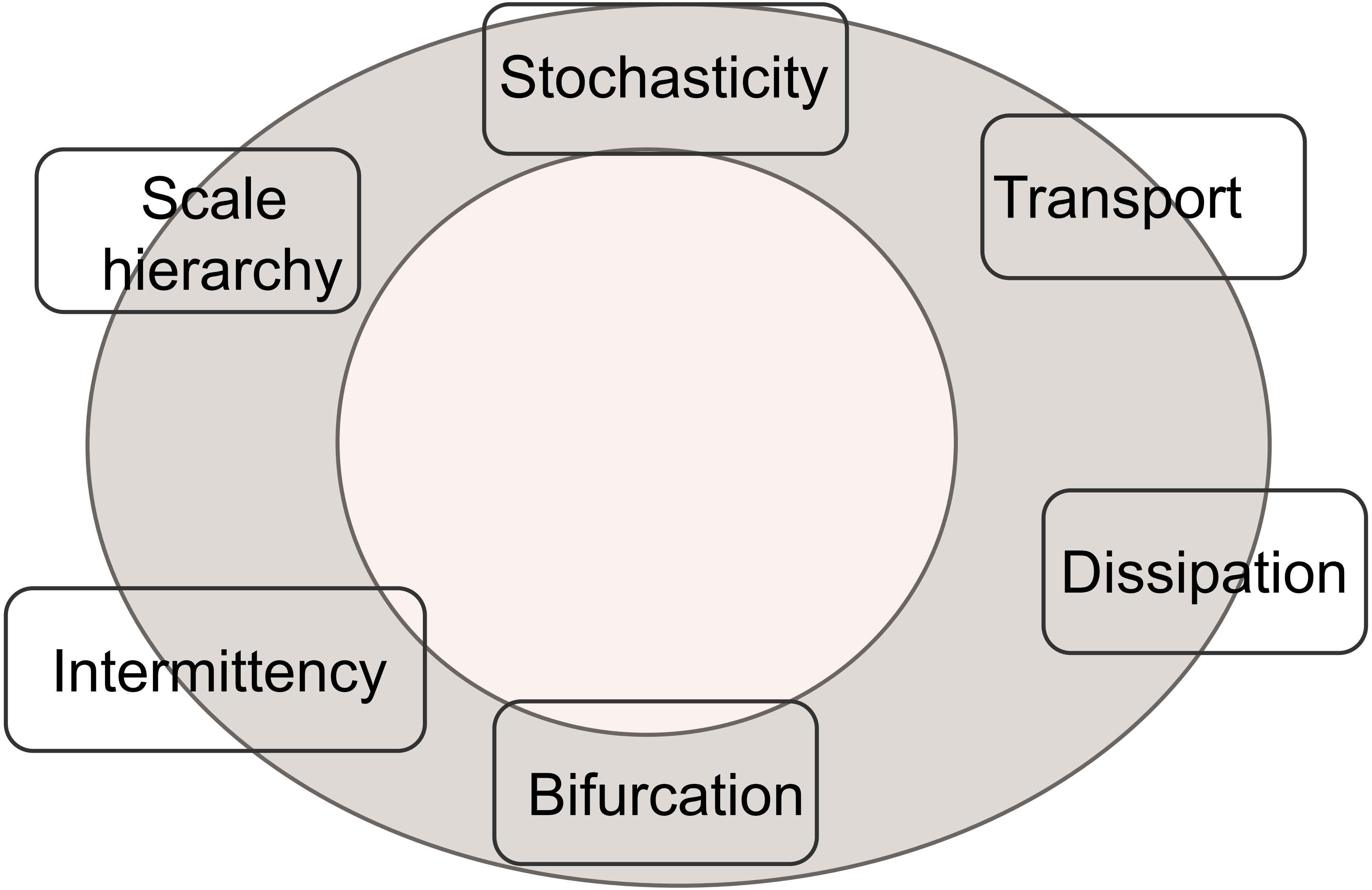
Class 2: the Mysteries of Turbulence

Physics of Turbulence

I am an old man now, and when I die and go to heaven there are two matters on which I hope for enlightenment. One is quantum electrodynamics, and the other is the turbulent motion of fluids. And about the former I am rather optimistic.

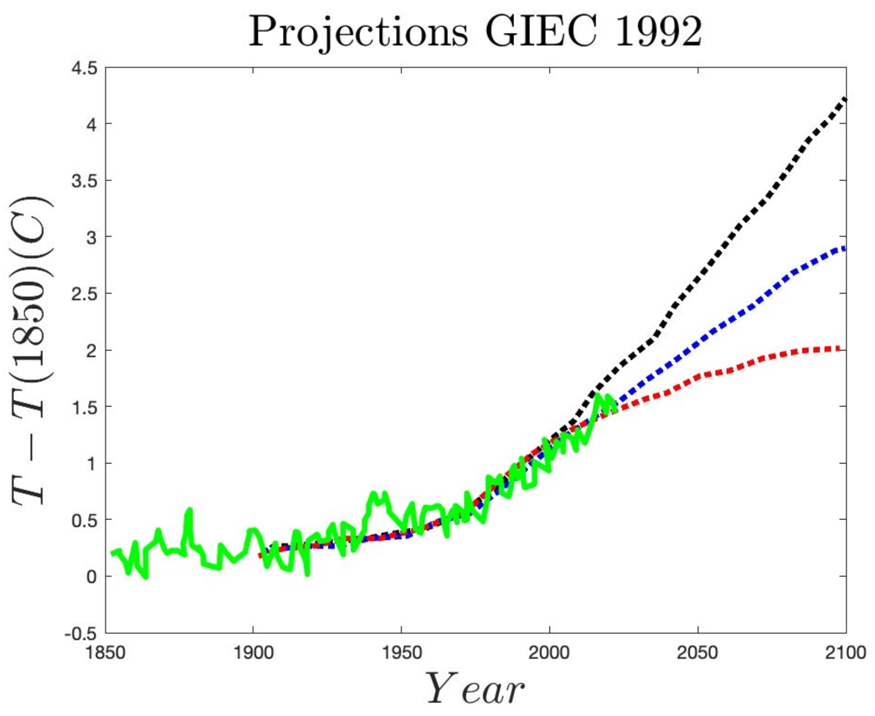
Sir Horace Lamb

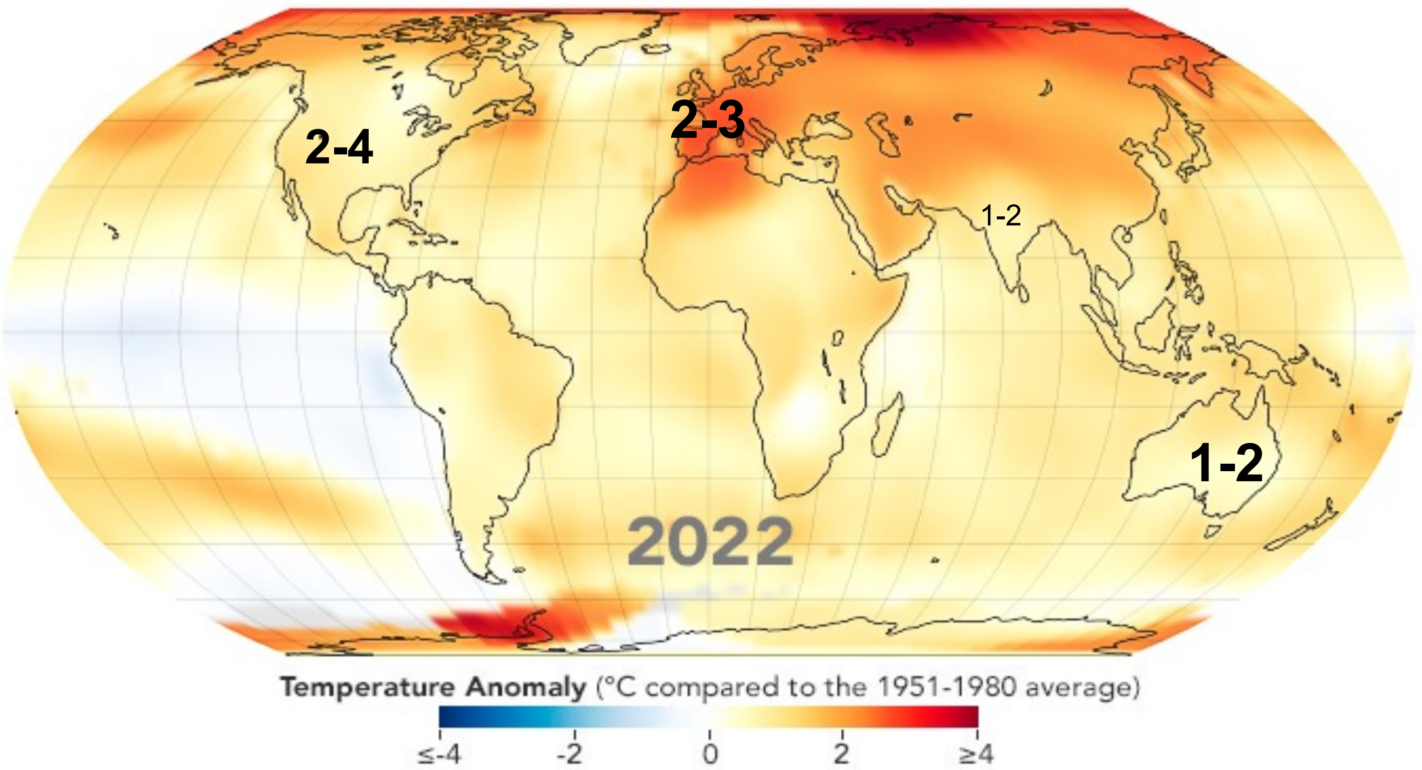

Stochasticity
Scale hierarchy
Transport
Dissipation
Intermittency
Bifurcation

Return

Stochasticity of turbulence

Mean vs fluctuations predictions

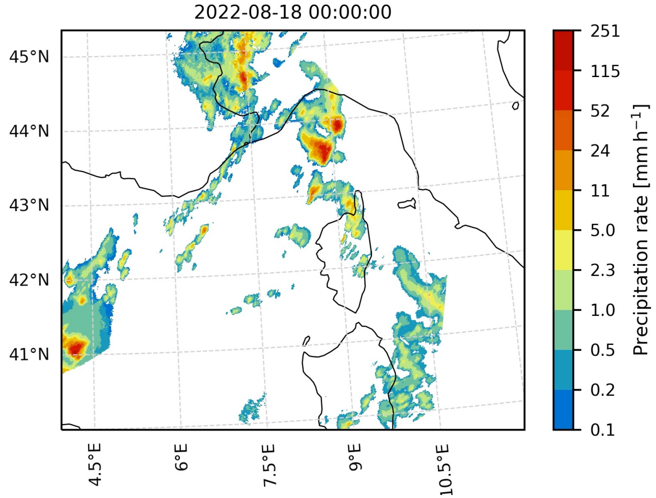


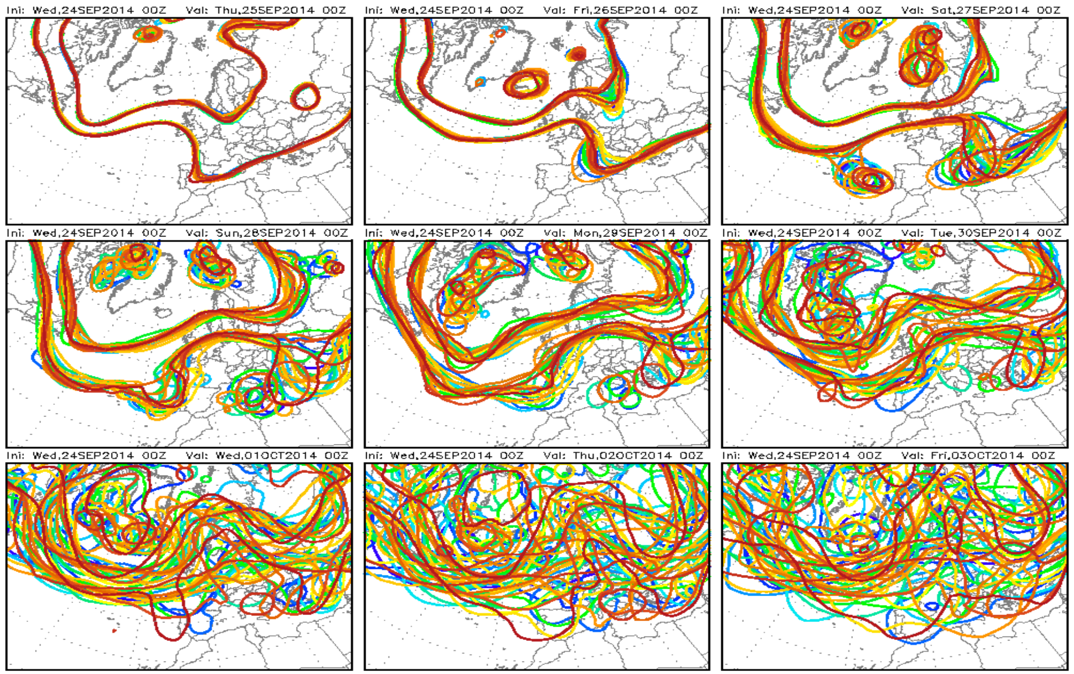


Climate, global: easy

Climate, global: less easy

Mean vs fluctuations predictions





Wheather= more difficult!

Where does it come from?

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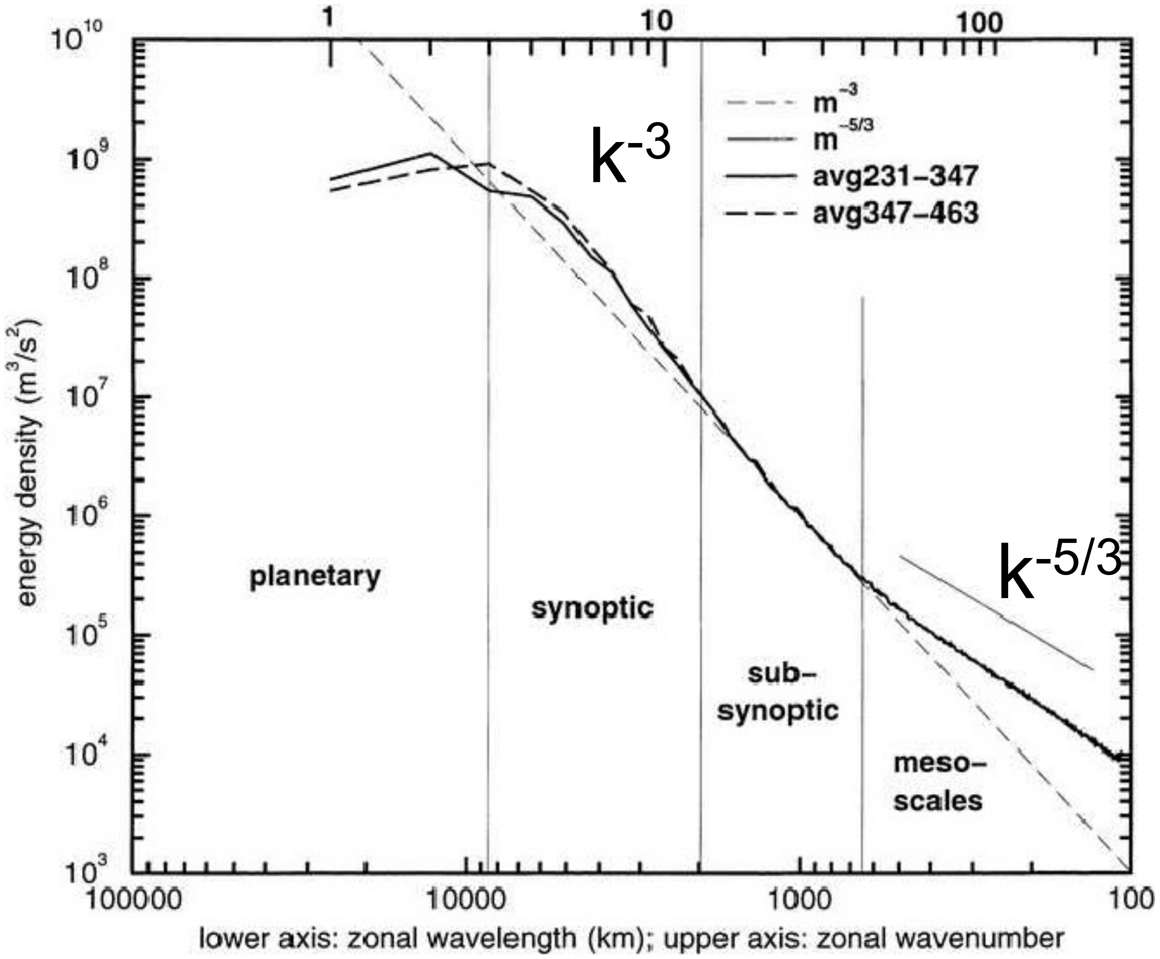
Scale hierachy of turbulence

The puzzling weather energy spectra



$$\Omega = k^2 E(k)$$

$$\Omega = k^{-1}$$

Regular

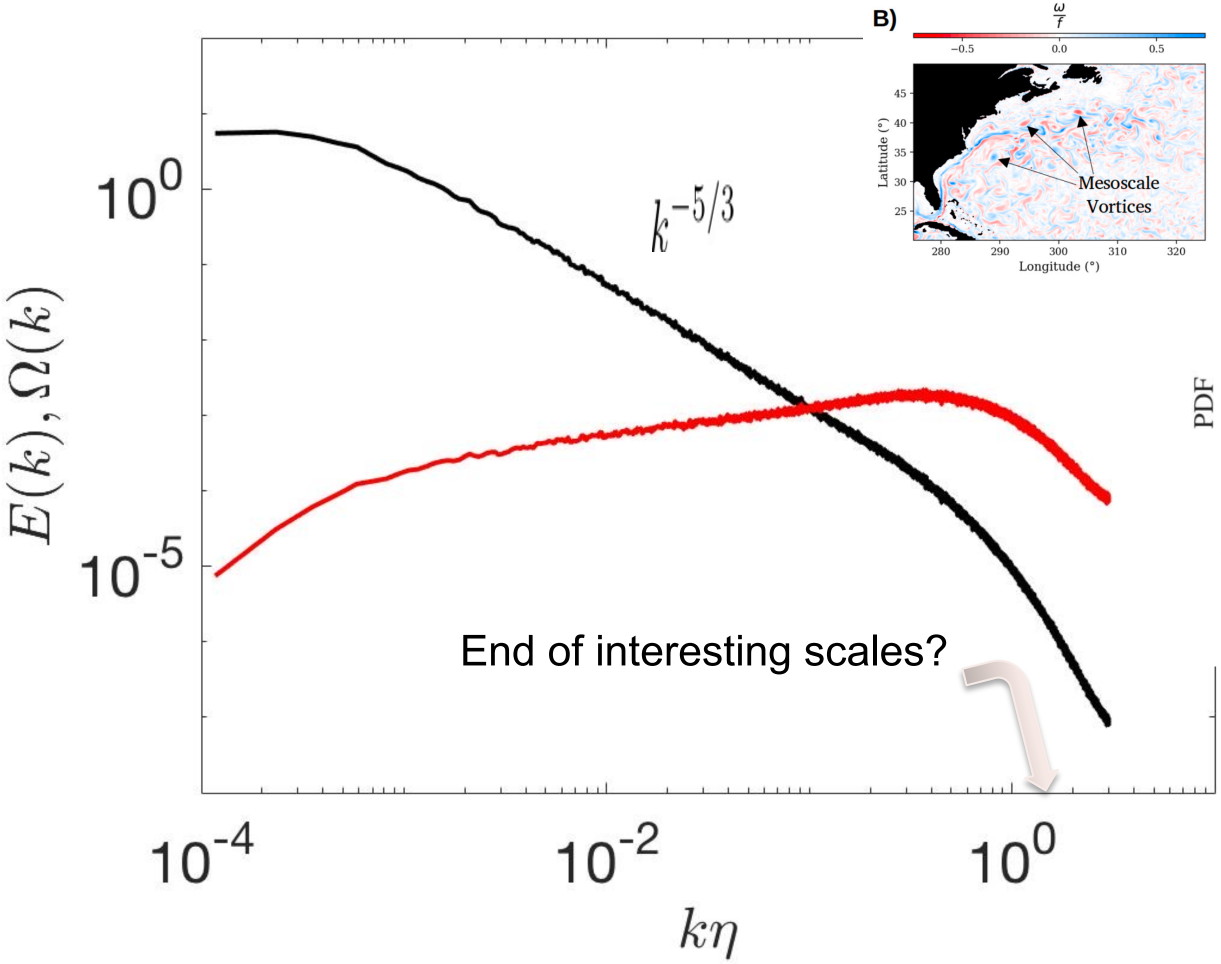
$$\Omega = k^{2/3}$$

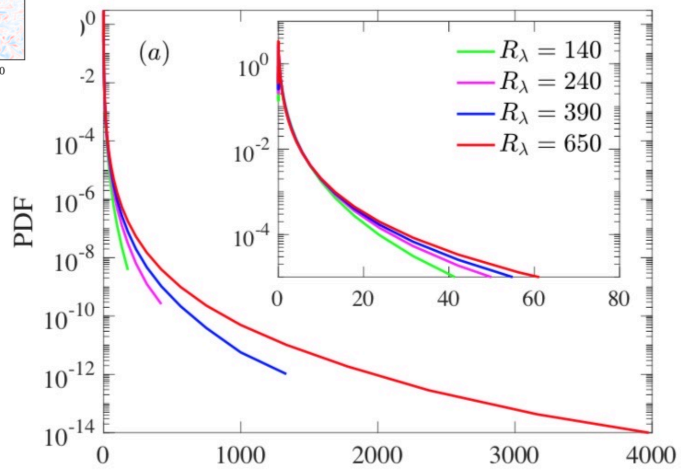
Rough

At small scale, enstrophy grows

Where does the difference come from?

Velocity gradients increase as Reynolds is increased!





Building of extreme velocity gradients at increasing Reynolds

Bhuaria et al, 2020

Indication for blow-up of velocity gradients in the inviscid limit!

Is there a problem?

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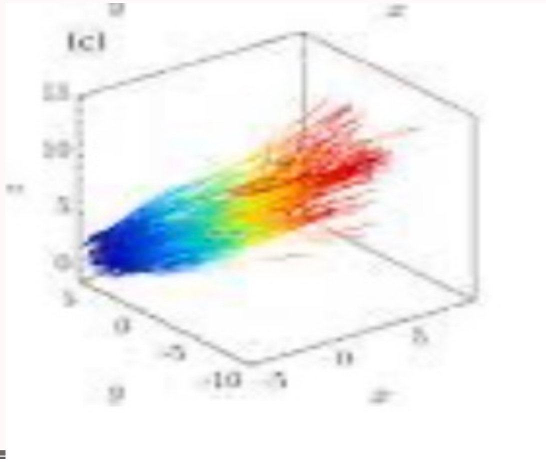
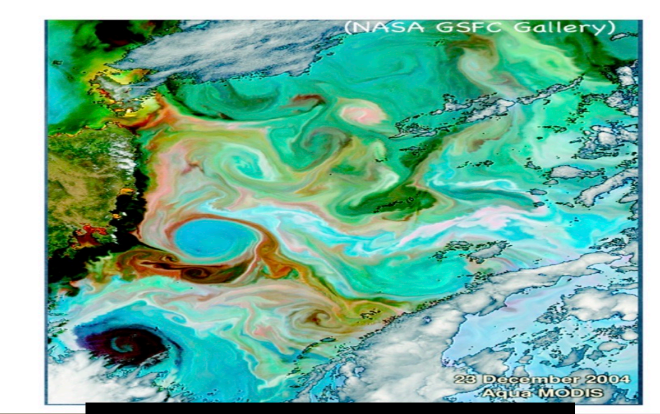
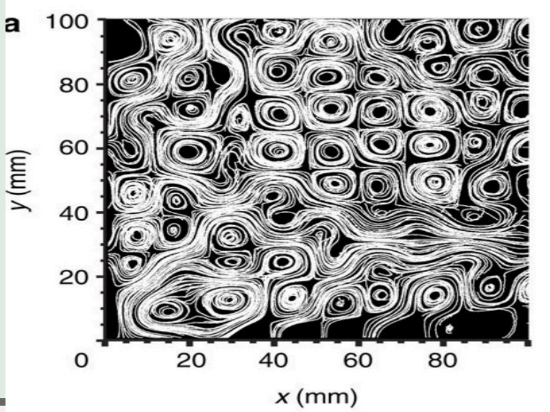
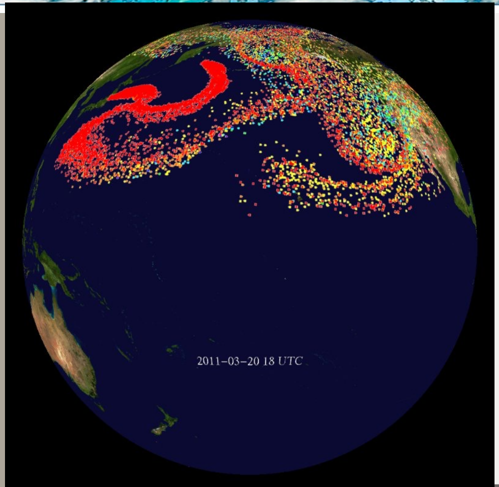
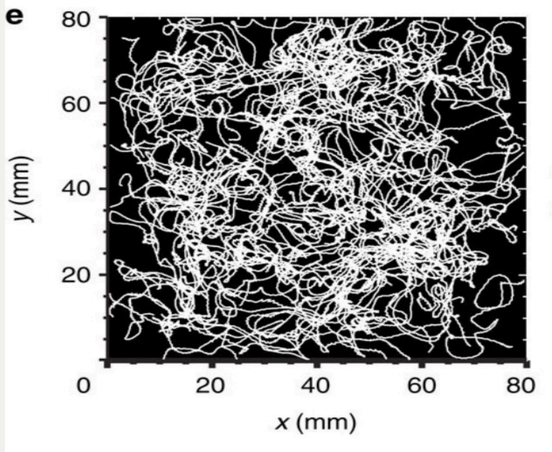
The transport regimes of turbulence

Type		Trajectories	$< (\delta x)^2 >$
BALLISTIC			$< \Delta x^2 > \sim (\sigma_u t)^2$ $< \delta R^2 > \sim R_0^{2/3} t^2$
CHAOTIC			$< \delta R^2 > \sim R_0^2 e^{\Lambda t}$
TURBULENT			$< \Delta x^2 > \sim \sigma_u^2 t$ $< \delta R^2 > \sim t^3$

Some mathematics

Chaotic dispersion

$$< (\delta x)^2 > \sim \exp(2\Lambda t)$$

$$\delta x(t) \sim \exp(\Lambda t)$$

$$\delta \dot{x}(t) \sim \Lambda \exp(\Lambda t)$$

$$\delta \dot{x} \sim \Lambda \delta x$$

Finite gradients

$$\partial_x v = \Lambda$$

Turbulent dispersion

$$< (\delta x)^2 >= \epsilon t^3$$

$$\delta x(t) \sim (\epsilon t^3)^{1/2}$$

$$\delta \dot{x}(t) \sim (\epsilon t)^{1/2}$$

$$\delta \dot{x} \sim (\epsilon \delta x)^{1/3}$$

Infinite gradients

$$\partial_x v = \infty$$

$$\partial_x v = \lim_{\delta x \to 0} \frac{\delta \dot{x}}{\delta x}$$

Some mathematics

Chaotic dispersion

$$< (\delta x)^2 >\sim \exp(2\Lambda t)$$

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$$\delta \dot{x}(t) \sim \Lambda \exp(\Lambda t)$$

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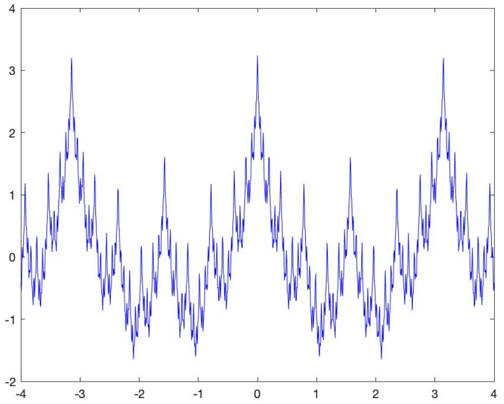
Finite gradients

$$\partial_x v = \Lambda$$



Turbulent dispersion

$$< (\delta x)^2 >= \epsilon t^3$$

$$\delta x(t) \sim (\epsilon t^3)^{1/2}$$
$$\delta \, \dot{x}(t) \sim (\epsilon t)^{1/2}$$

$$\delta \dot{x} \sim (\epsilon \delta x)^{1/3}$$

Infinite gradients

$$\partial_x v = \infty$$

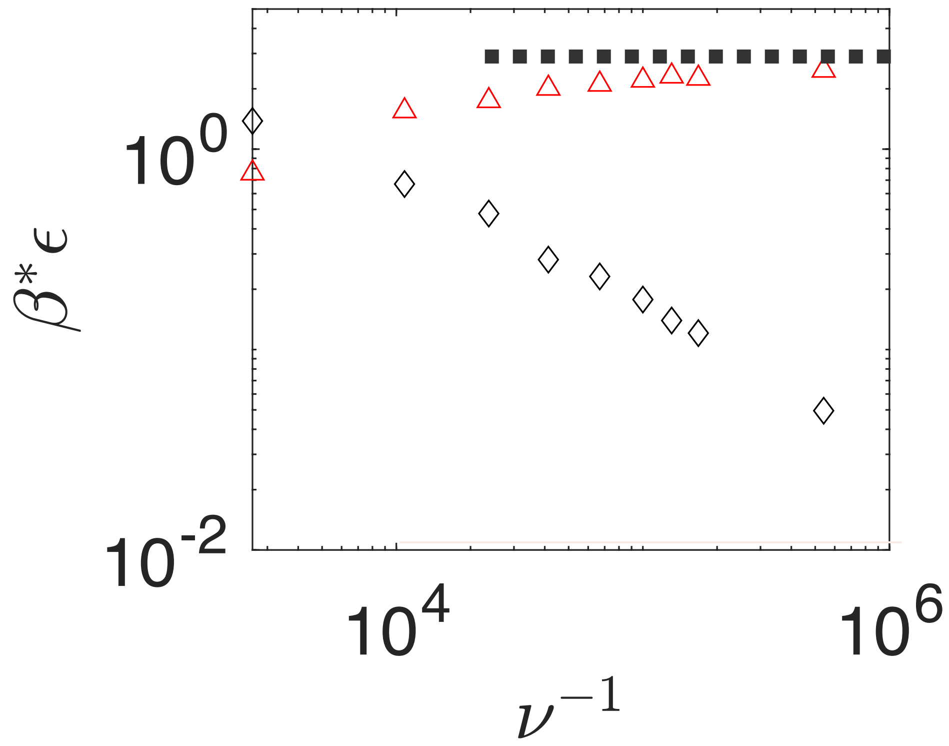
Is there a problem?

Return

Dissipation of turbulence

Dissipation, singularities and anomaly

Saturation of $$\epsilon = \nu\langle(\nabla u)(\nabla u)^{\perp}\rangle$$



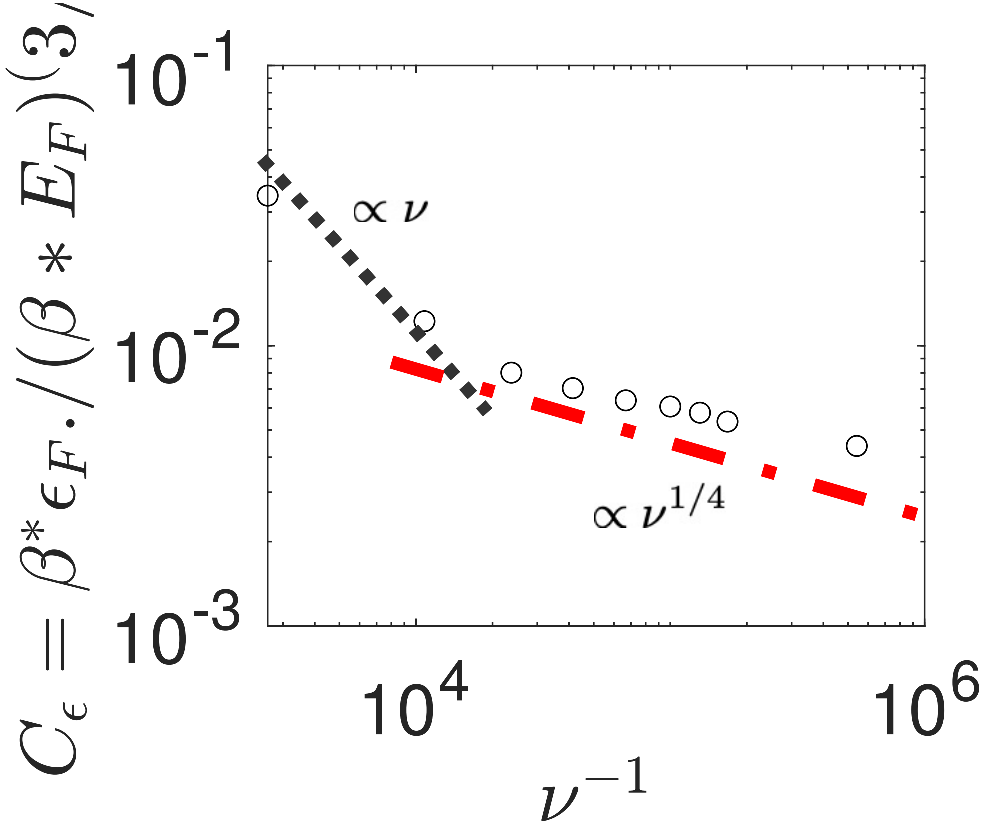
$$\rightarrow \boxed{\langle(\nabla u)(\nabla u)^{\perp}\rangle \to \infty}$$

C1: *Spontaneous time reversal symmetry breaking*

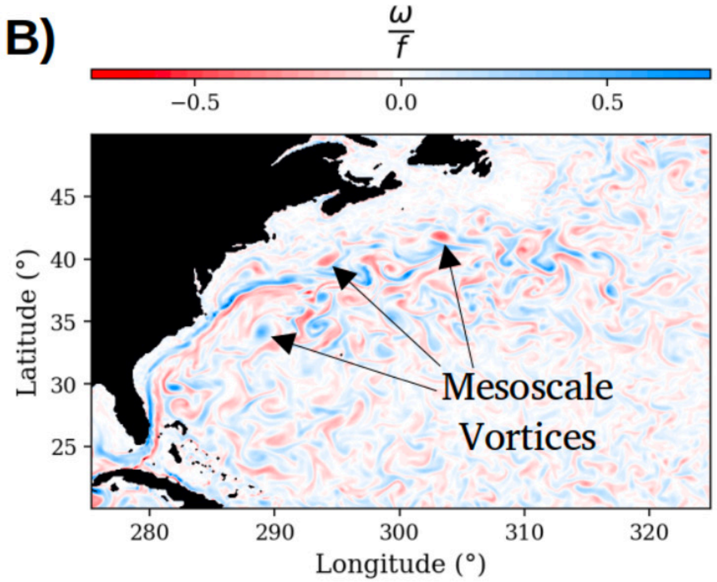
C2: $$\boxed{\partial_x v = \infty}$$

Coherent structures and Dissipation

Non-dimensional dissipation

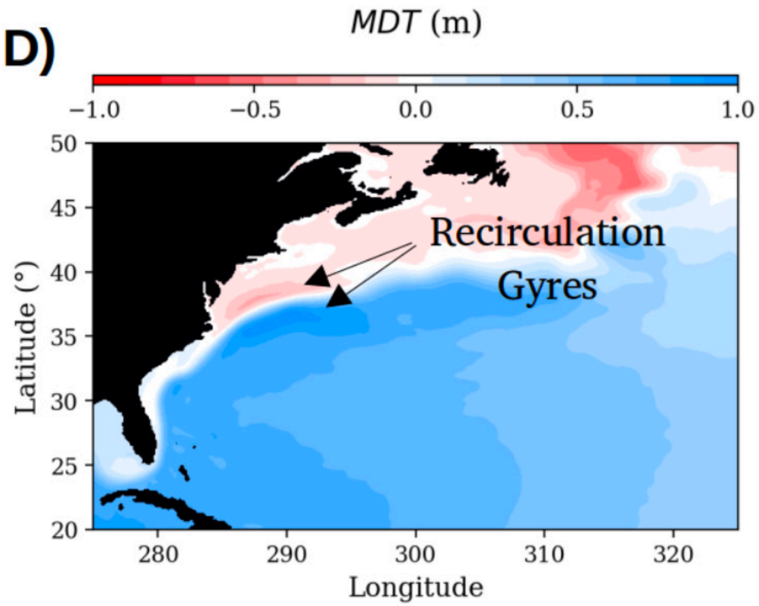


Miller PhD Thesis



Random

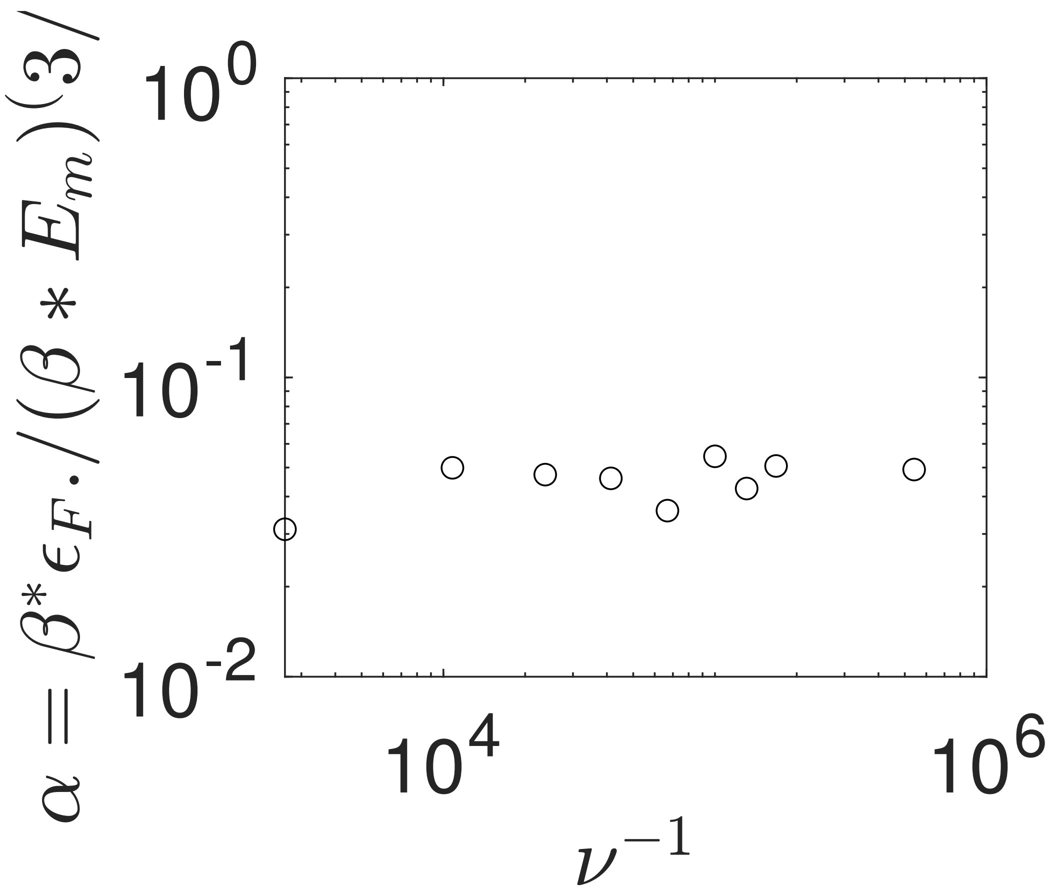
Vortices exerts a constant friction on the large scale



Mean

Coherent structures and Dissipation

Friction



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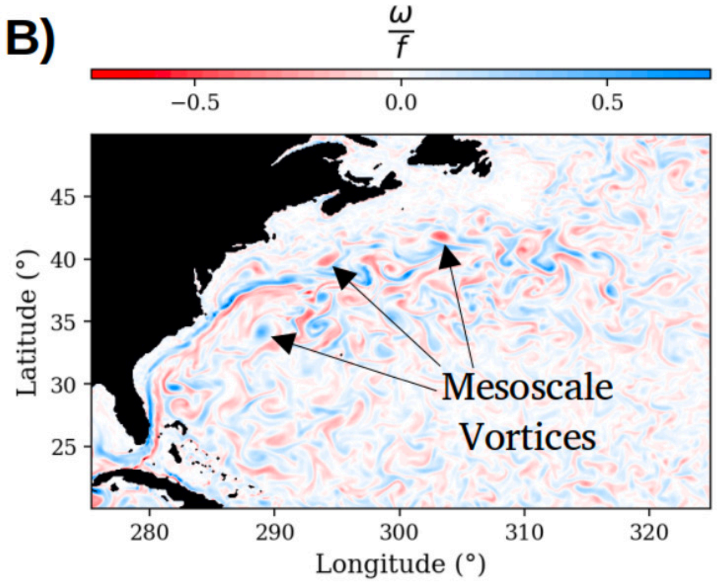

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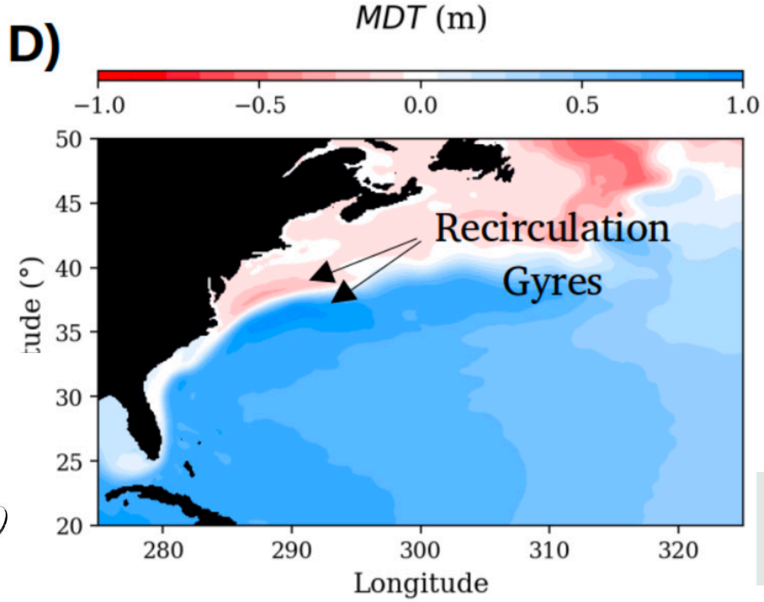
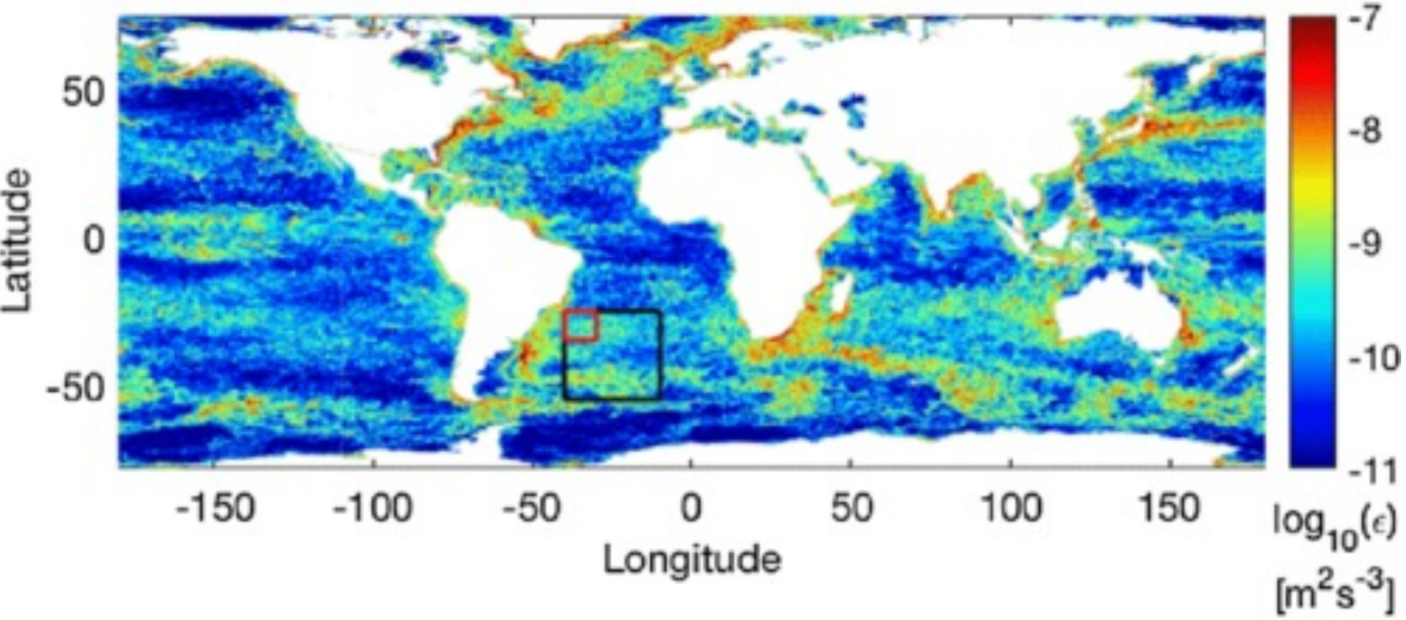


Mean

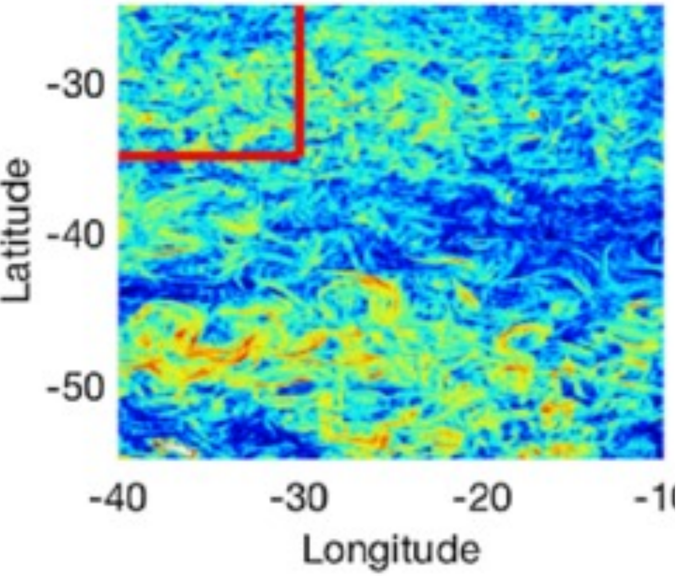
Return

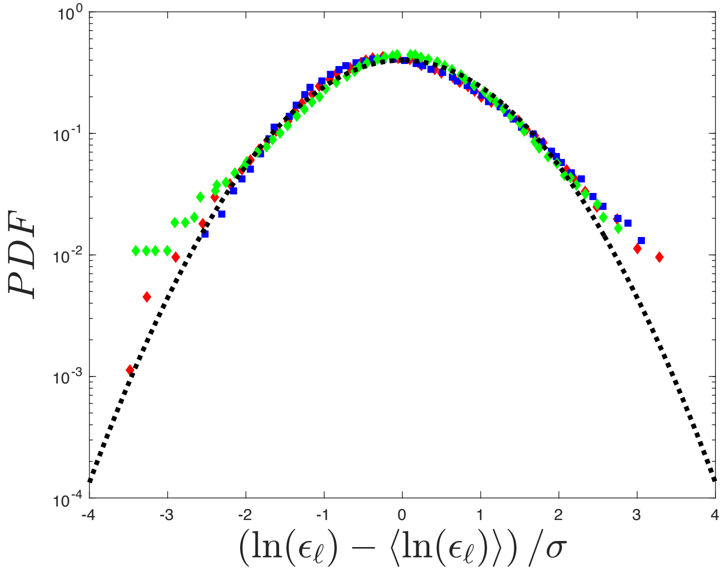
The diverging nature of dissipation?



$$< \epsilon^p > \sim < \epsilon >^p \exp(\mu p(p-1))$$

$$\epsilon_\infty = \lim_{p \to \infty} \frac{< \epsilon^{p+1} >}{< \epsilon^p >} \sim \exp(2\mu p) \to \infty$$

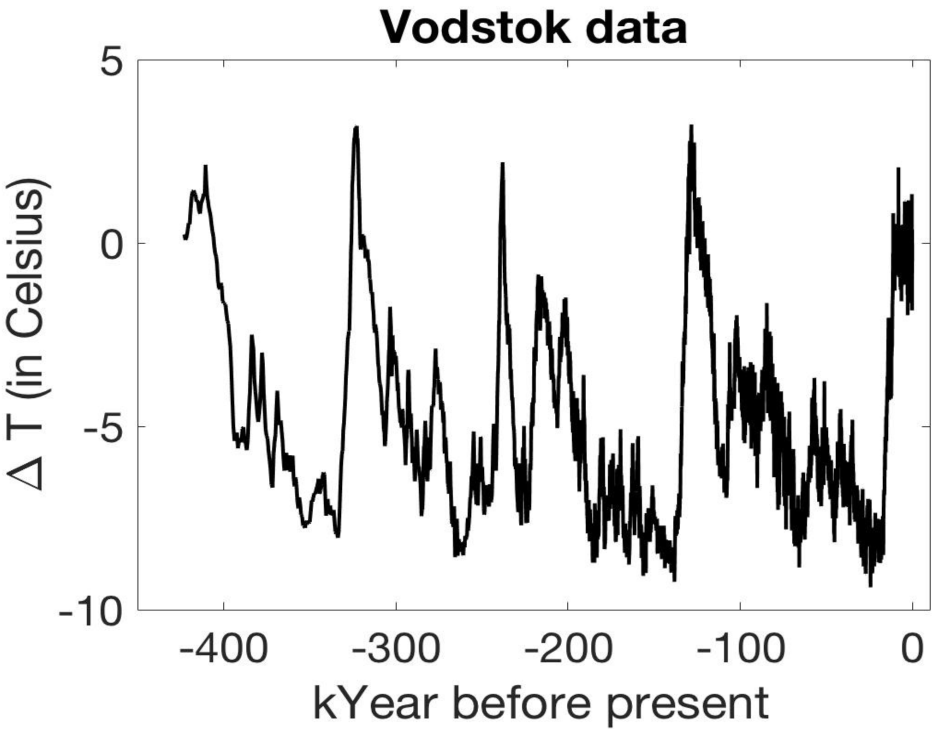


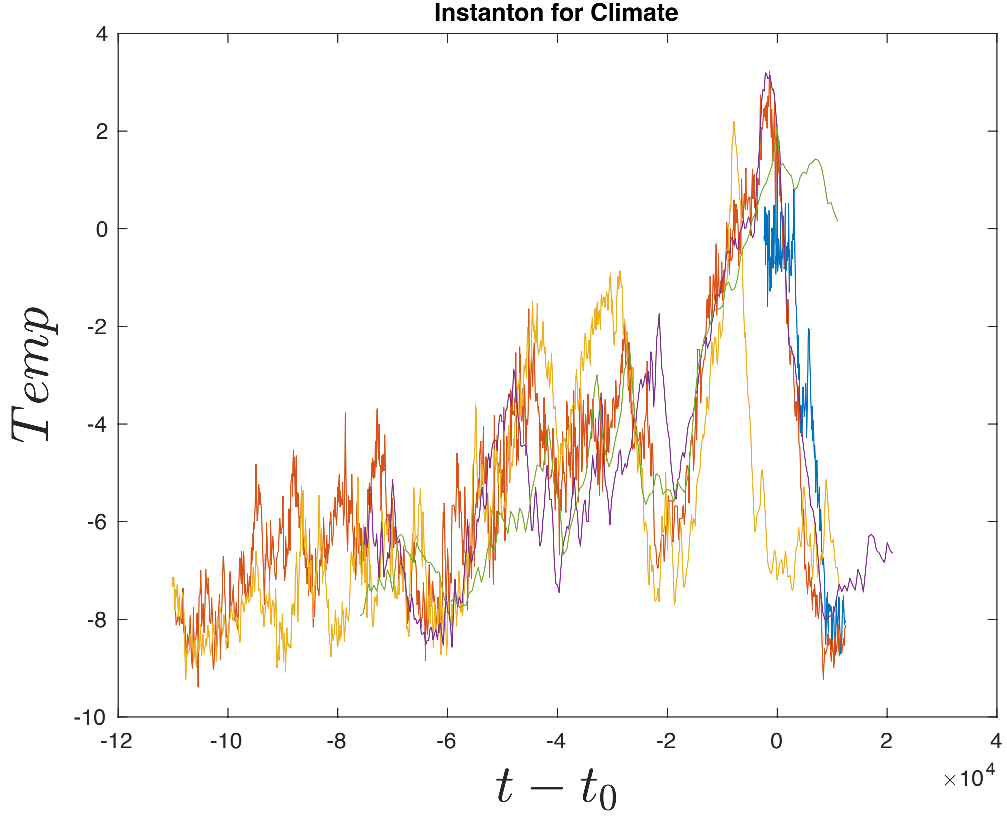


Return

Bifurcations in turbulence

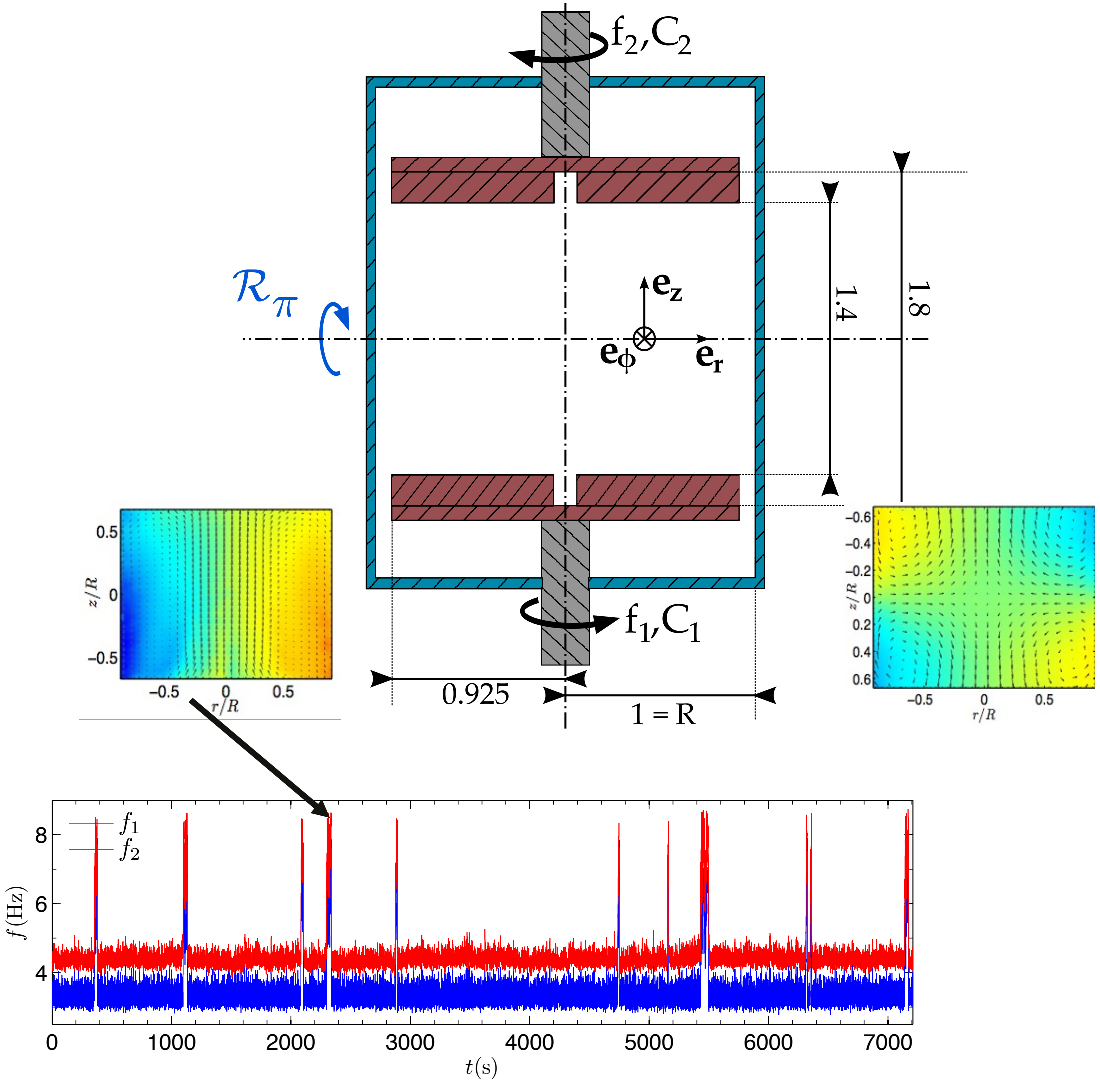
Reproducibility of transition: Instantons





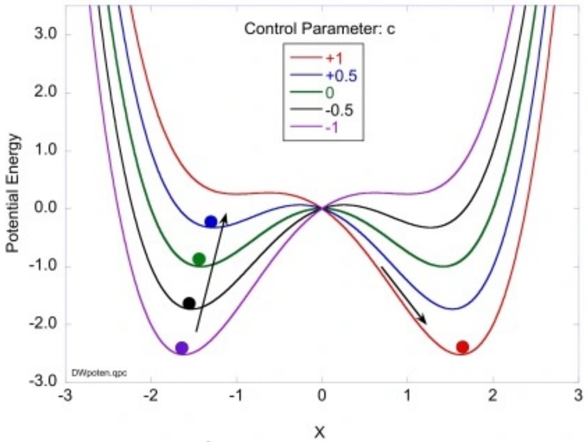
Why is this stochastic transition reproducible?

Role of symmetry breaking variable

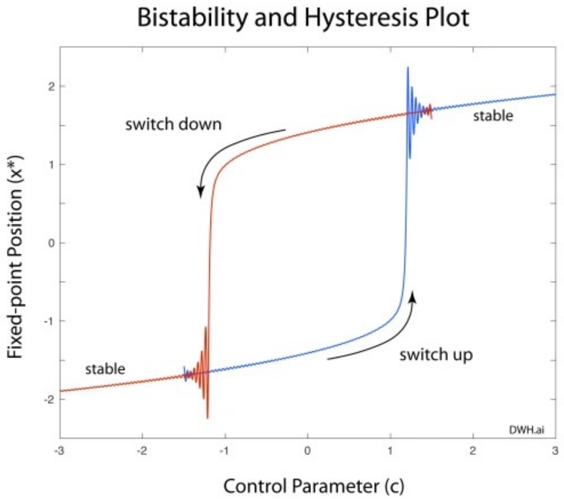


Abruptness of transition

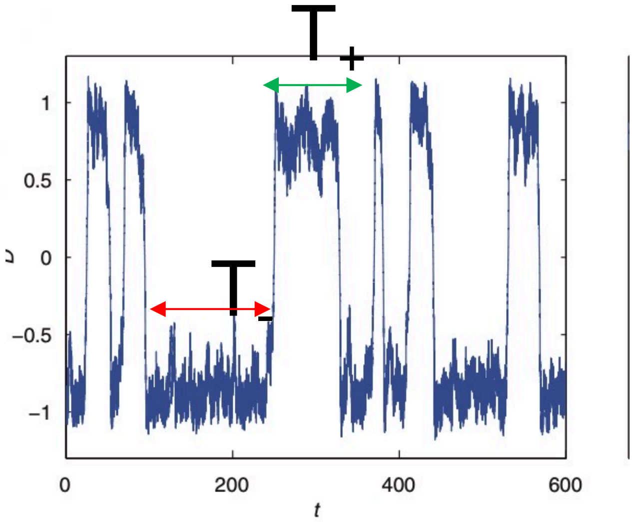
Paragdimatic model: bifurcation with noise

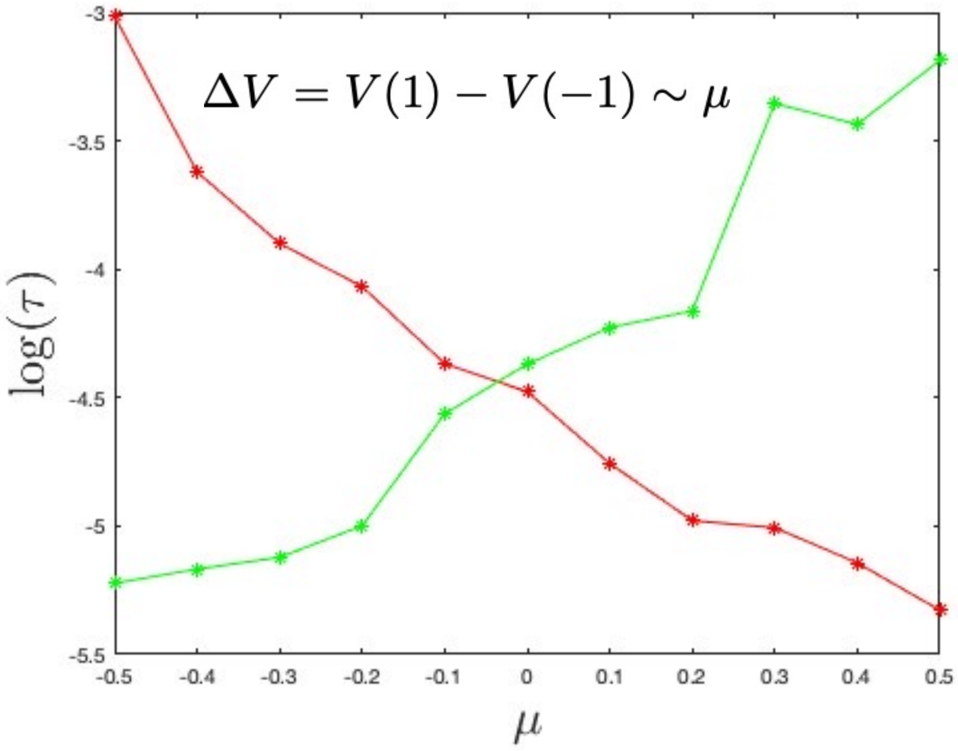


$$V(x) = \frac{1}{4}x^4 - x^2 - cx$$



$$dx = -\partial_x V\,dt + \sigma dW$$





Persistence law for large enough barrier

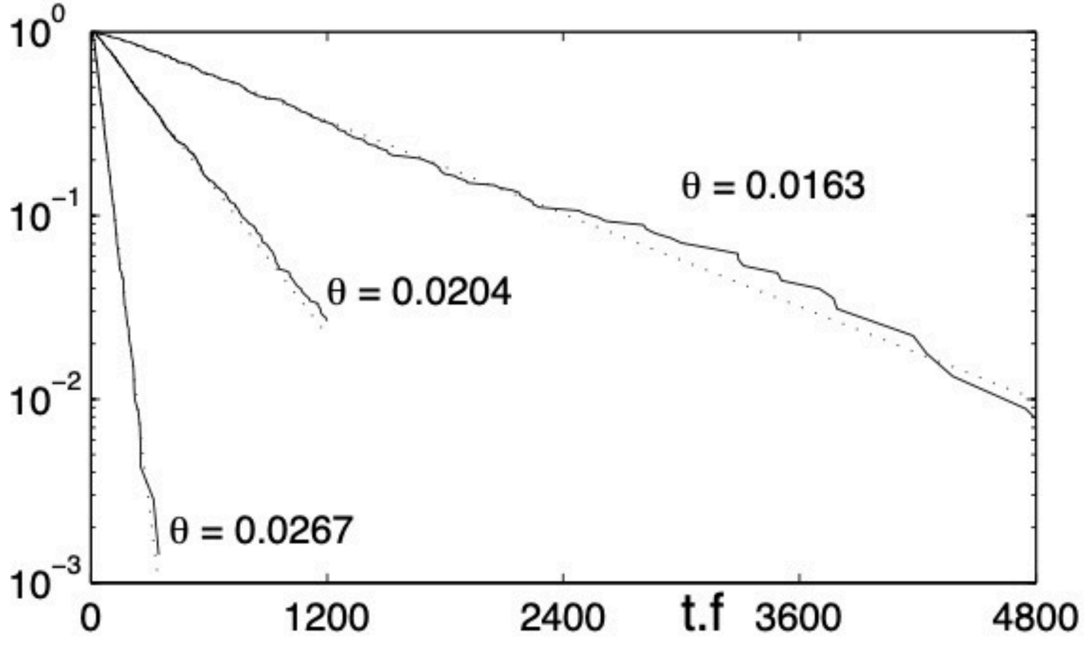
$$p(T) \sim \exp(-T/\tau)$$

Arhenius law

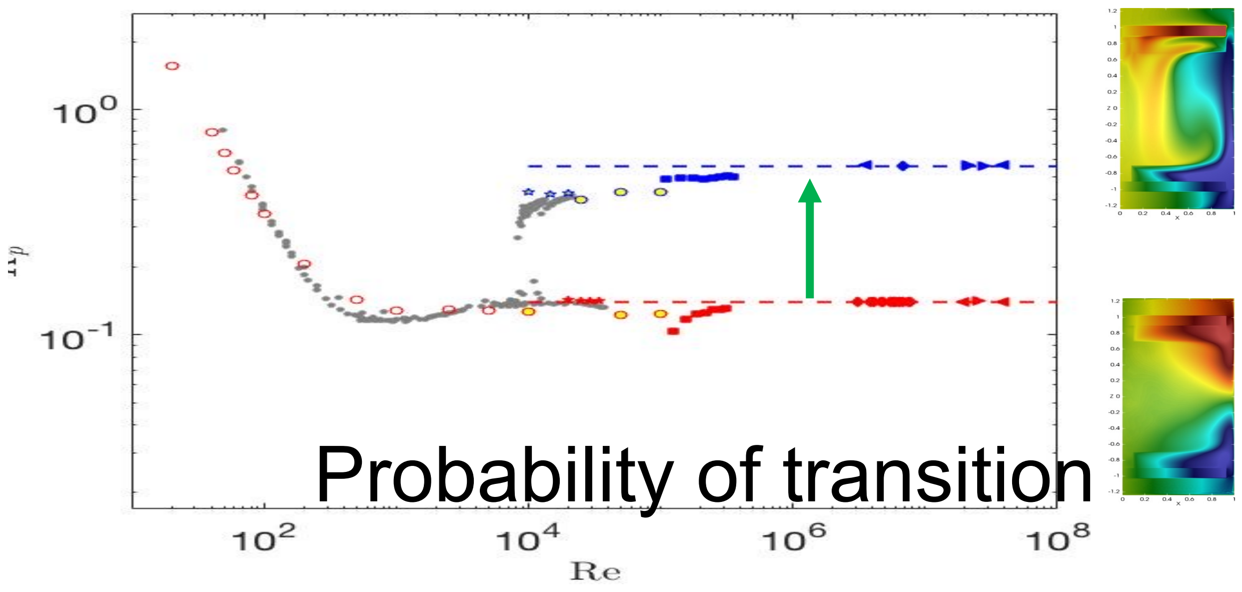
$$\tau \sim \exp(-\Delta V)/\sigma^2)$$

Persistence in VK transition

Symmetry parameter: θ



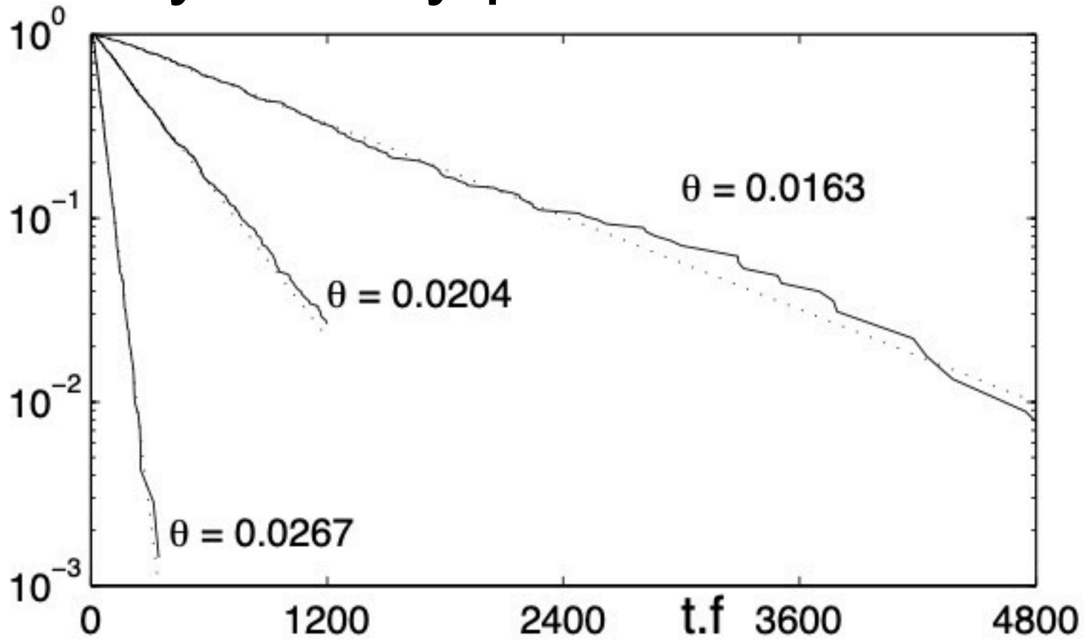
Ravelet et al et al, PRL (2004)



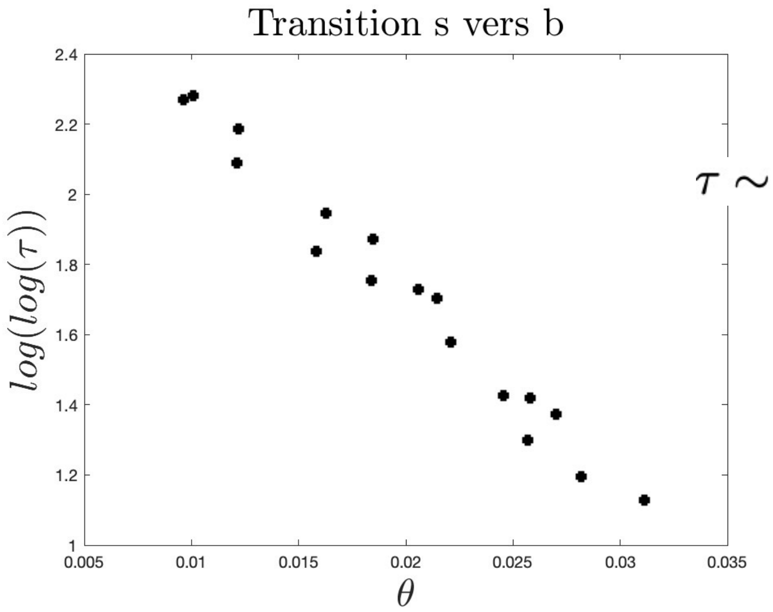
Ravelet et al et al, JFM (2008)
Saint-Michel et al, Physics of Fluids 26, 125109 (2014)
Cappanera et al, Computers&Fluids (2021)

Persistence in VK transition

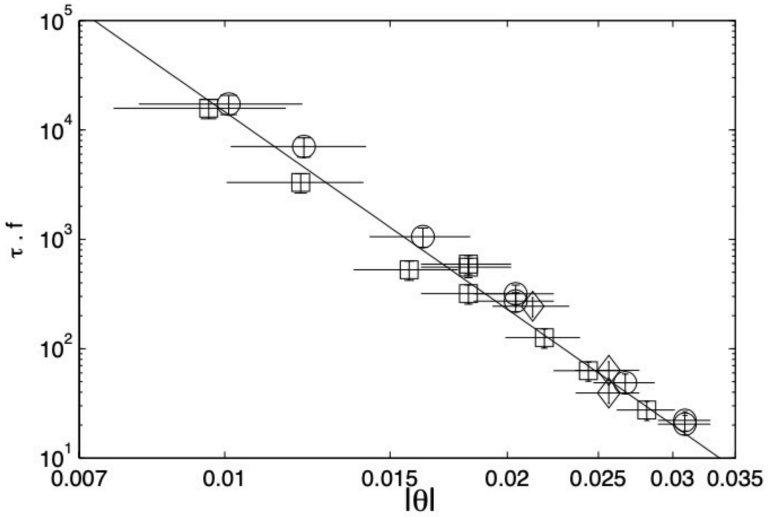
Symmetry parameter: θ



Ravelet et al et al, PRL (2004)



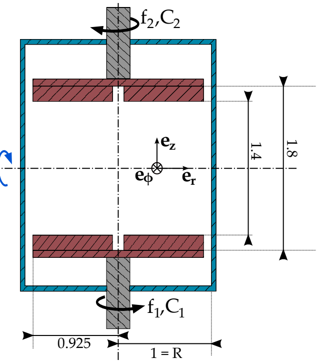
$$\tau \sim \exp(-\exp(\theta))$$

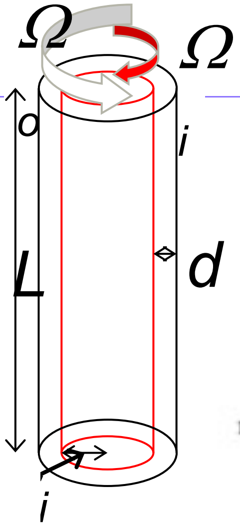


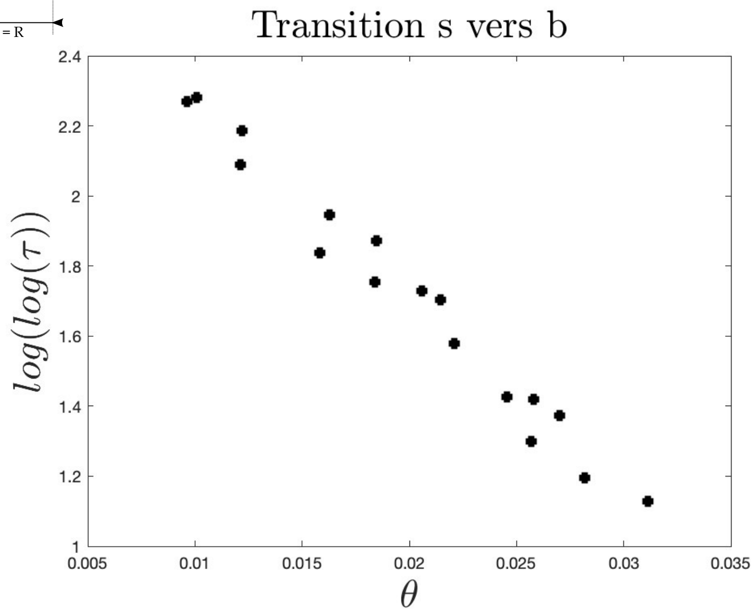
$$\tau \sim \theta^{-6}$$

Exponential law valid
Ahrenius law not valid

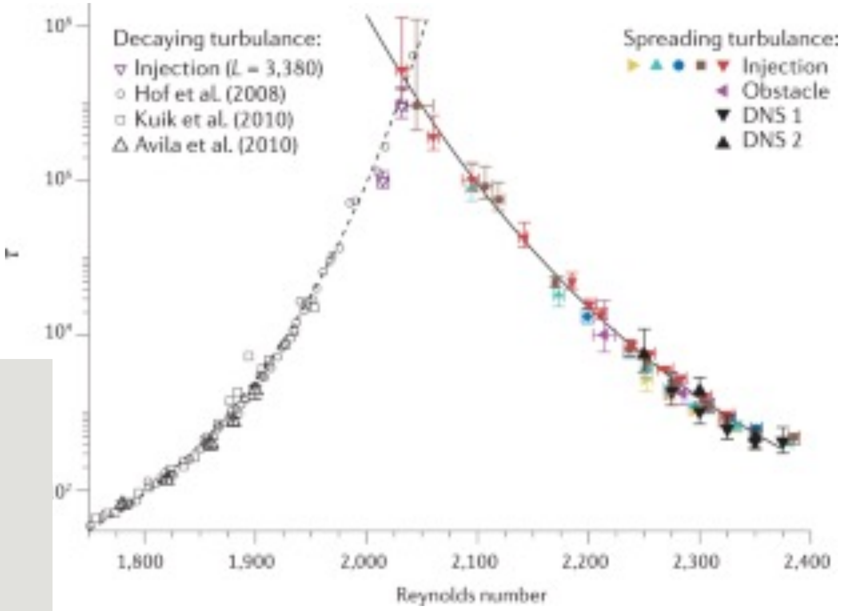
Super-exponential life times?







Link with rare events…
What kind?

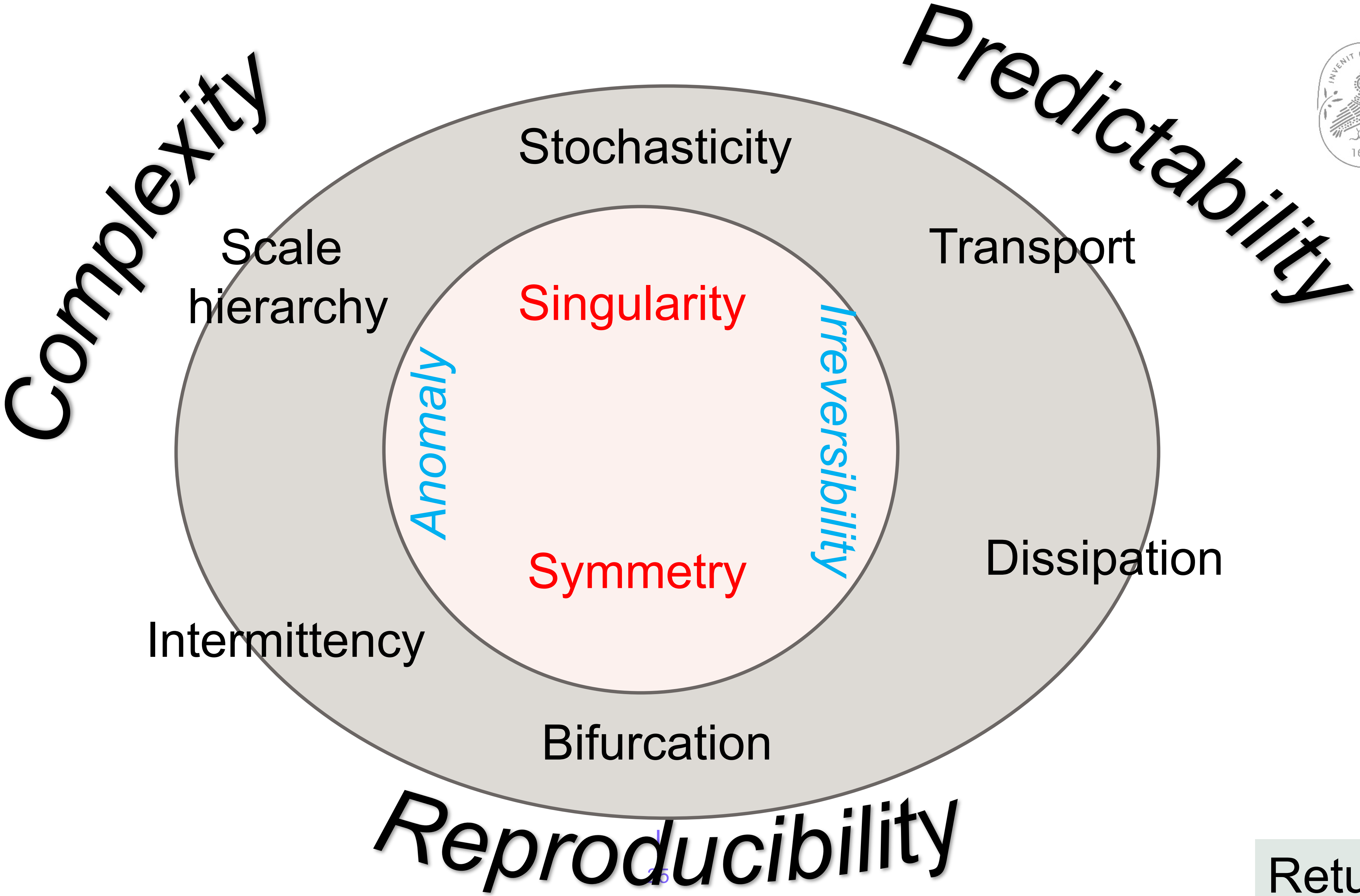


$$\tau \sim \exp(-\exp(\theta))$$

Cf Gumble
(distribution of extremes)

$$\sim \exp(-\exp(Re))$$

Return


Complexity
Predictability
Reproducibility
Stochasticity
Scale hierarchy
Transport
Dissipation
Intermittency
Bifurcation
Singularity
Symmetry
Anomaly
Irreversibility

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