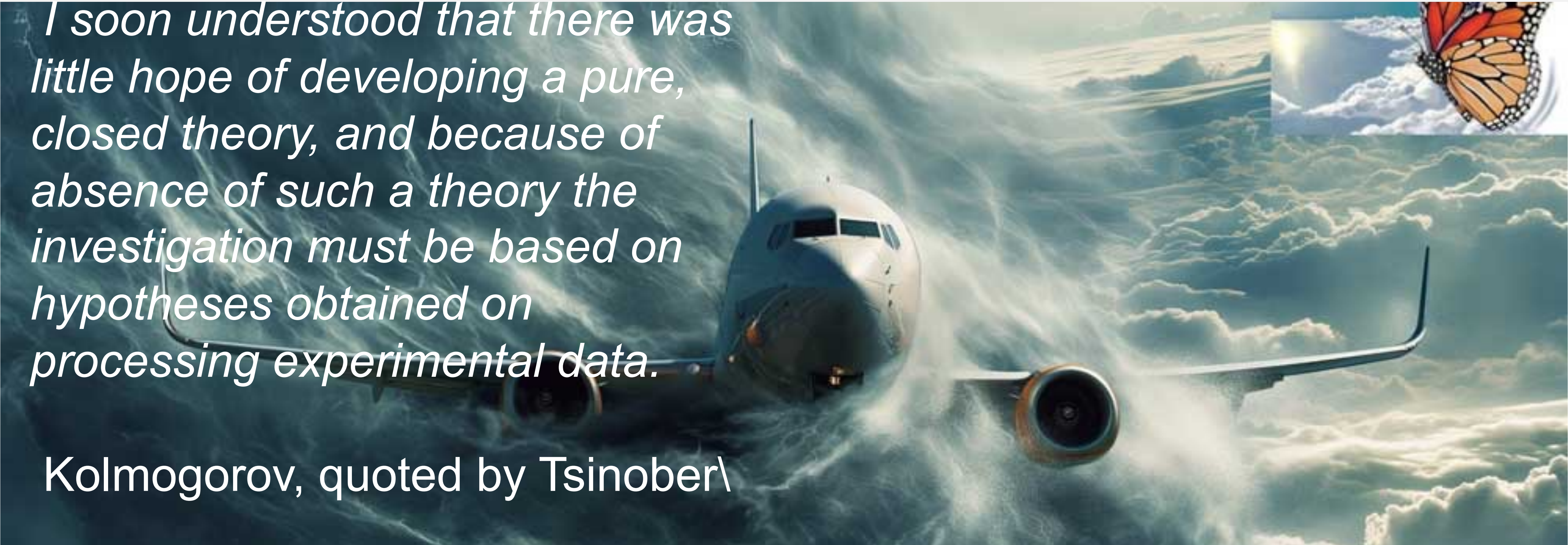
Class 1: the empirical laws of Turbulence

Physics of Turbulence



I soon understood that there was little hope of developing a pure, closed theory, and because of absence of such a theory the investigation must be based on hypotheses obtained on processing experimental data.

Kolmogorov, quoted by Tsinober\

*Définition: T**urbulence** describes the state of a fluid (liquid or gas) in which velocity is in a **swirling** state.*

Vortices in the Universe…

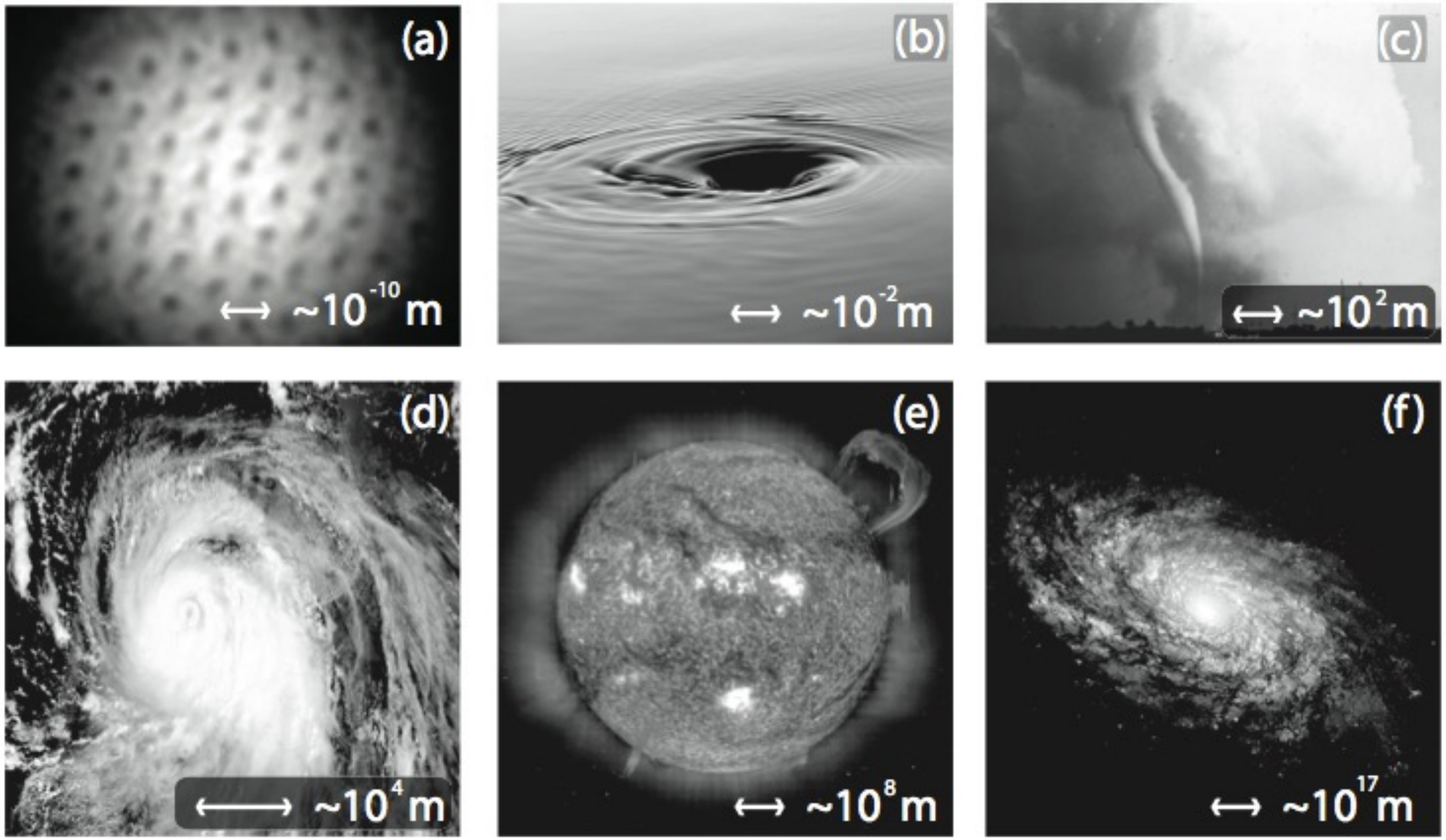
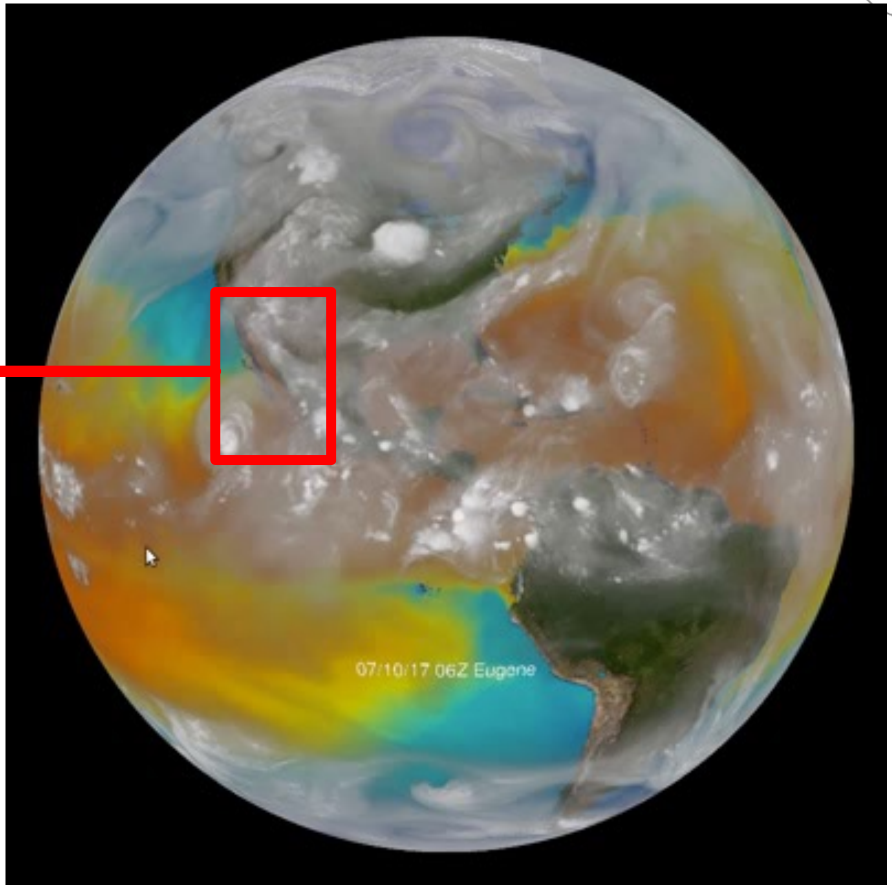


Figure 1.1: Vortices affect fluid behavior on all scales. (a) quantum vortices in a superfluid [130] (b) bathtub vortex [152] (c) tornado [109] (d) hurricane [106] (e) sun spot vortices [110] (f) spiral galaxy [105] (numbers approximate)

R. Fuchs, PhD thes

Turbulences or vortices?



Why turbulence is famous (practice)…

Why turbulence is famous (maths)…

PDE's of fluid dynamics

Burgers Equation

$$\partial_t u + u\partial_x u = \nu\partial_{xx} u$$

(Navier-Stokes without vorticity)

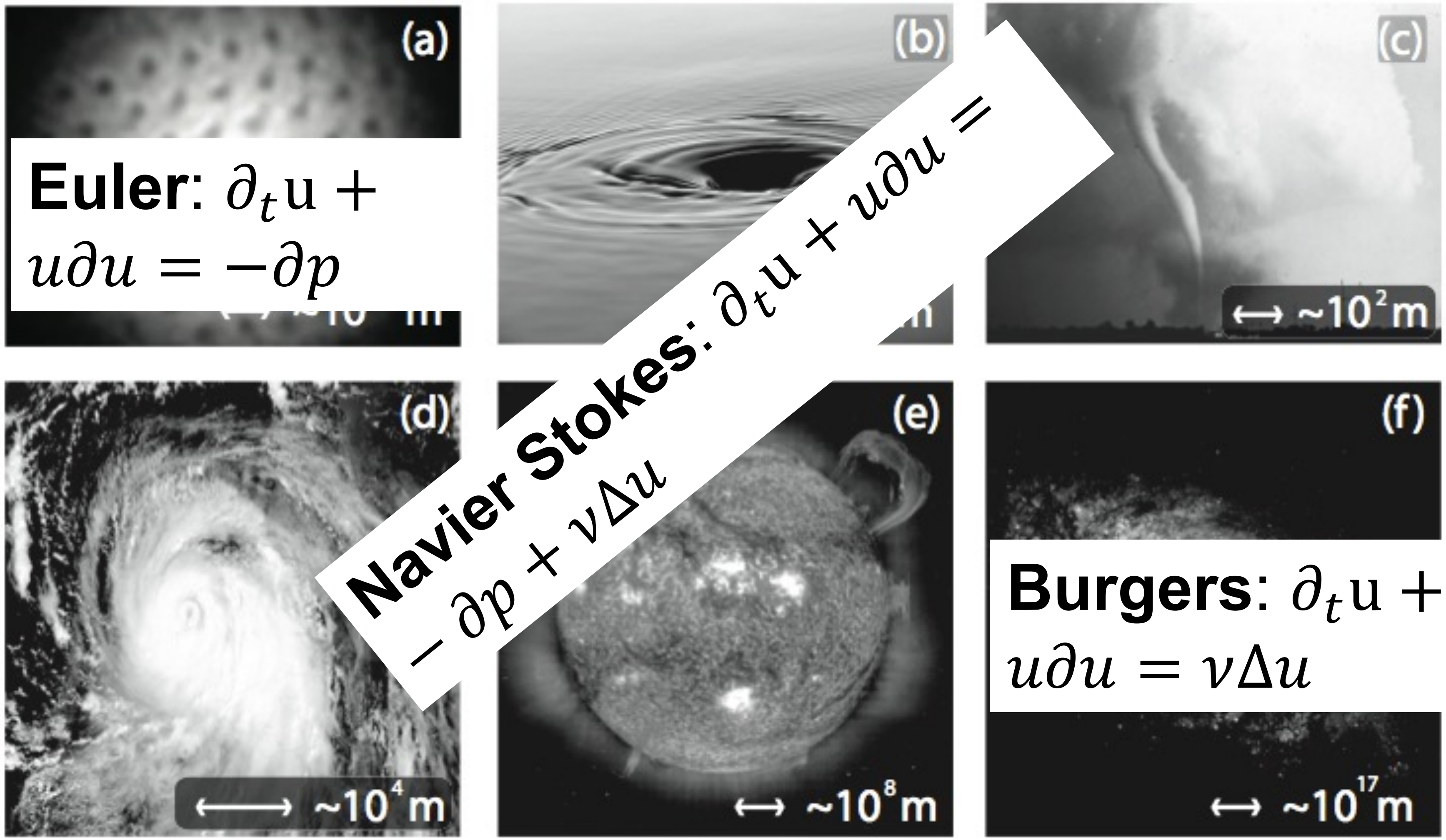
Navier-Stokes Equations

$$\partial_t u + u \cdot \nabla u = -\nabla p + \nu\Delta u$$

Euler Equations

$$\partial_t u + u \cdot \nabla u = -\nabla p$$

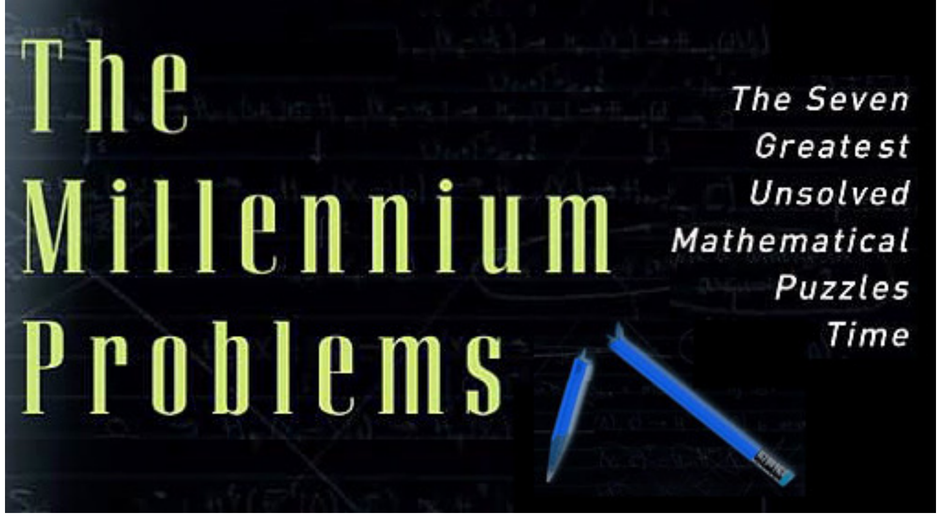
(Navier-Stokes without viscosity)

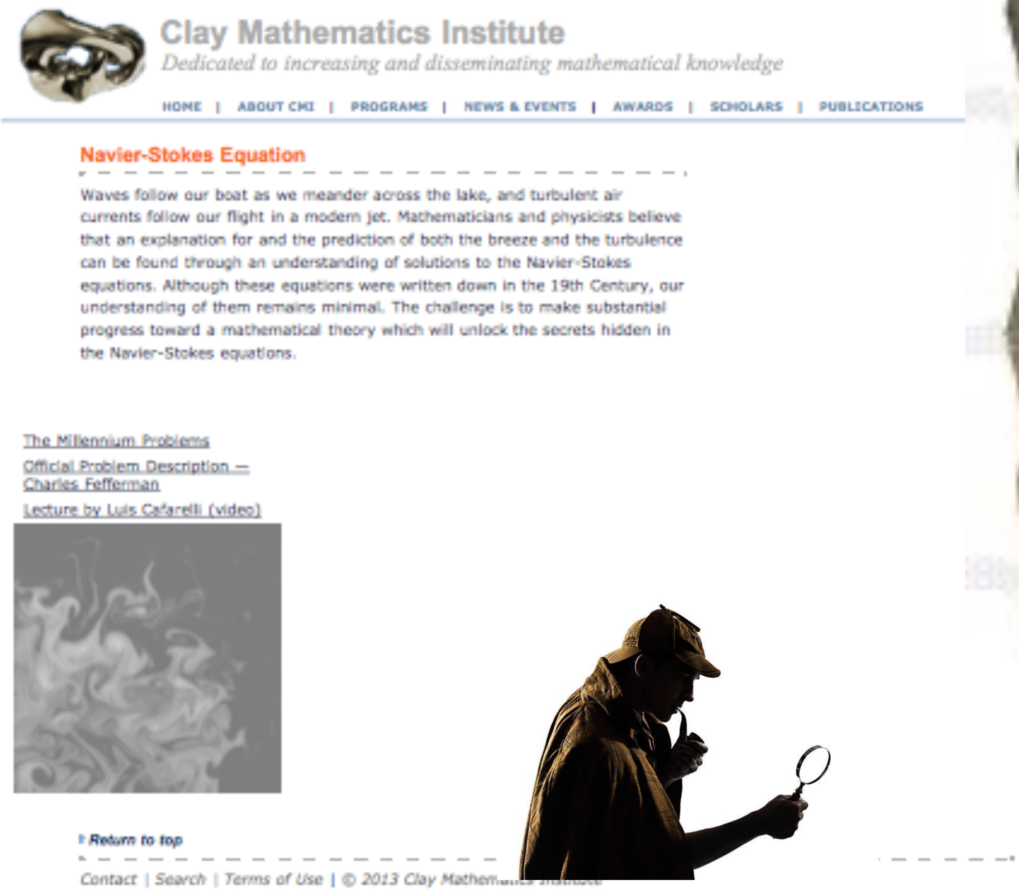


Euler: $\partial_t \mathrm{u} + u\partial u = -\partial p$

Navier Stokes: $\partial_t \mathrm{u} + u\partial u = -\partial p + \nu\Delta u$

Burgers: $\partial_t \mathrm{u} + u\partial u = \nu\Delta u$





Clay Mathematics Institute
Dedicated to increasing and disseminating mathematical knowledge

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Navier-Stokes Equation

Waves follow our boat as we meander across the lake, and turbulent air currents follow our flight in a modern jet. Mathematicians and physicists believe that an explanation for and the prediction of both the breeze and the turbulence can be found through an understanding of solutions to the Navier-Stokes equations. Although these equations were written down in the 19th Century, our understanding of them remains minimal. The challenge is to make substantial progress toward a mathematical theory which will unlock the secrets hidden in the Navier-Stokes equations.

The Millennium Problems
Official Problem Description — Charles Fefferman
Lecture by Luis Cafarelli (video)

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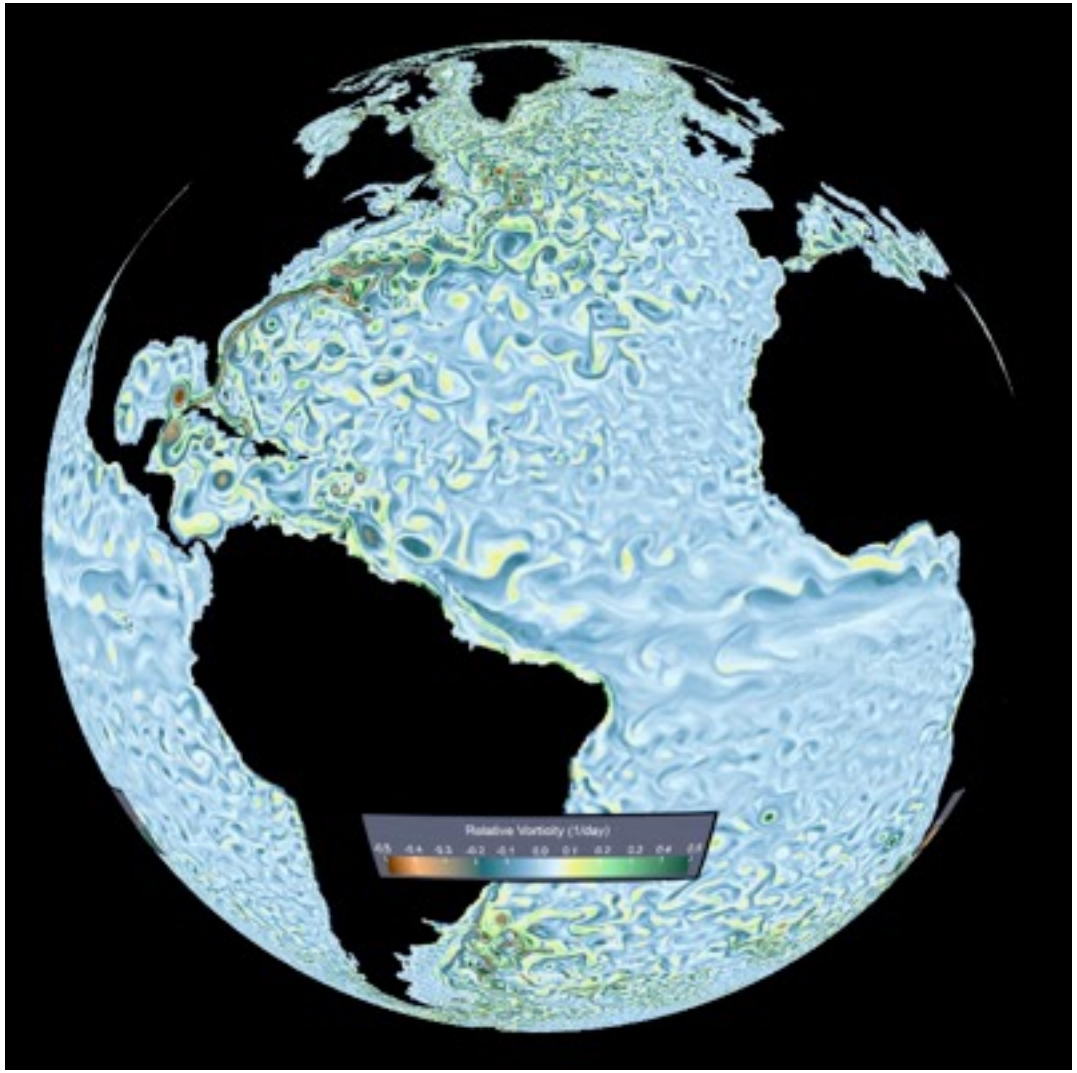
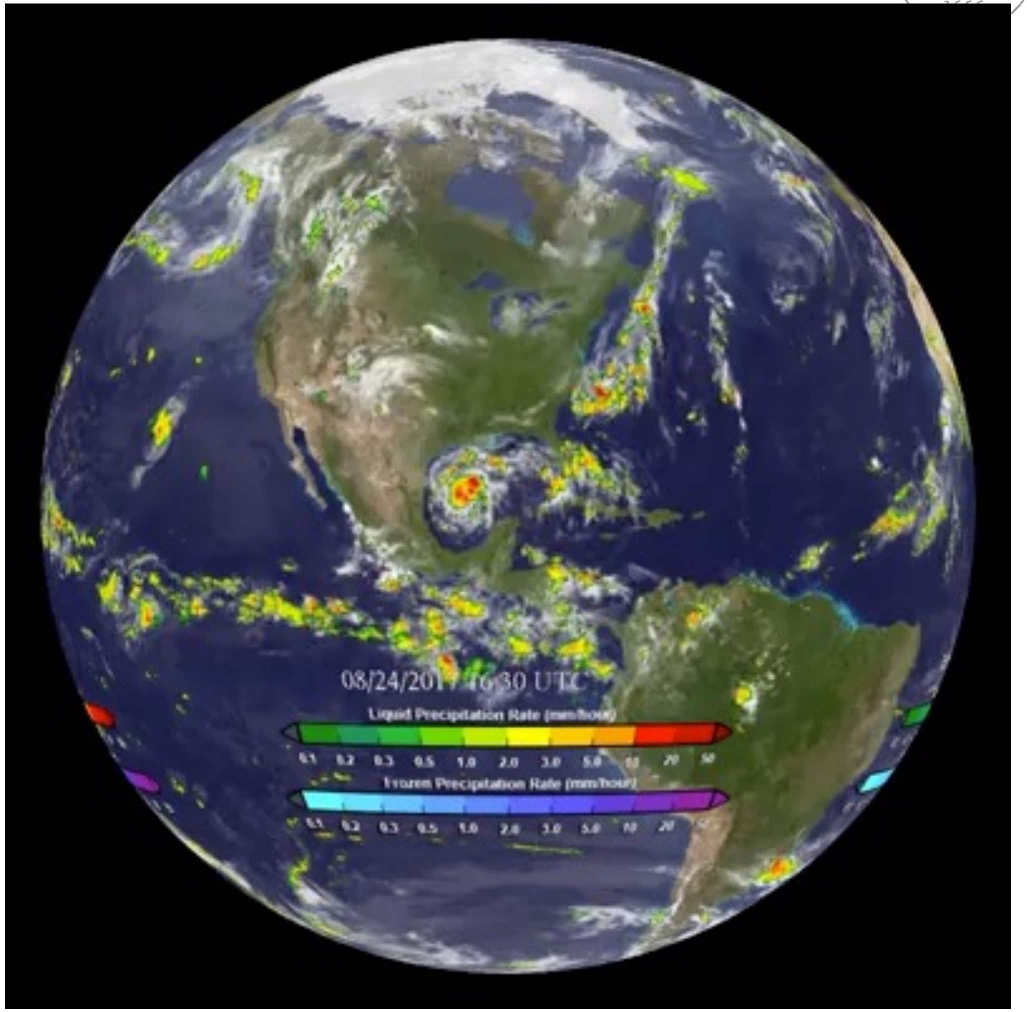
Proof of existence of singularities

Observation: The fluid envelops of the Earth



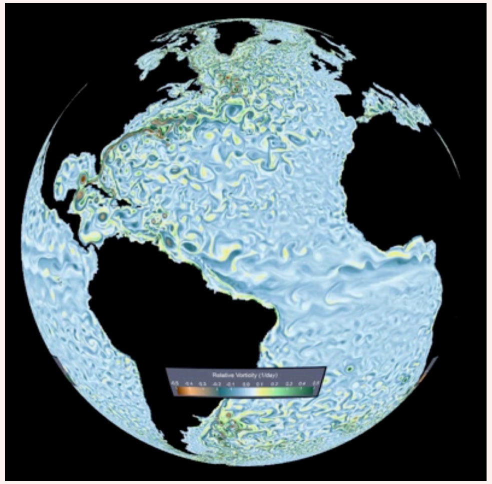


Observation: -> Empirical laws of turbulence

Stochasticity

Scale hierarchy

Transport



Dissipation

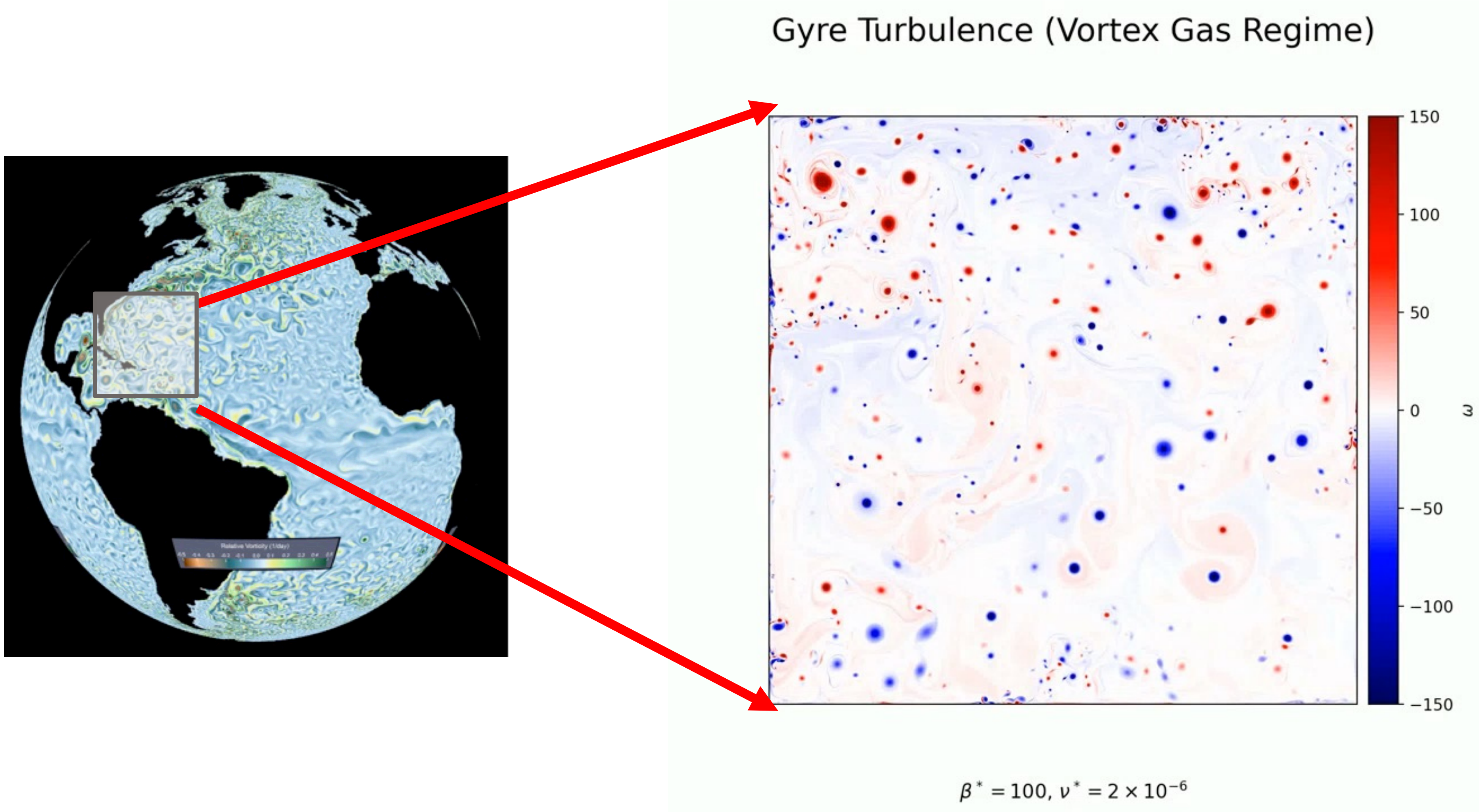
Intermittency

Bifurcation

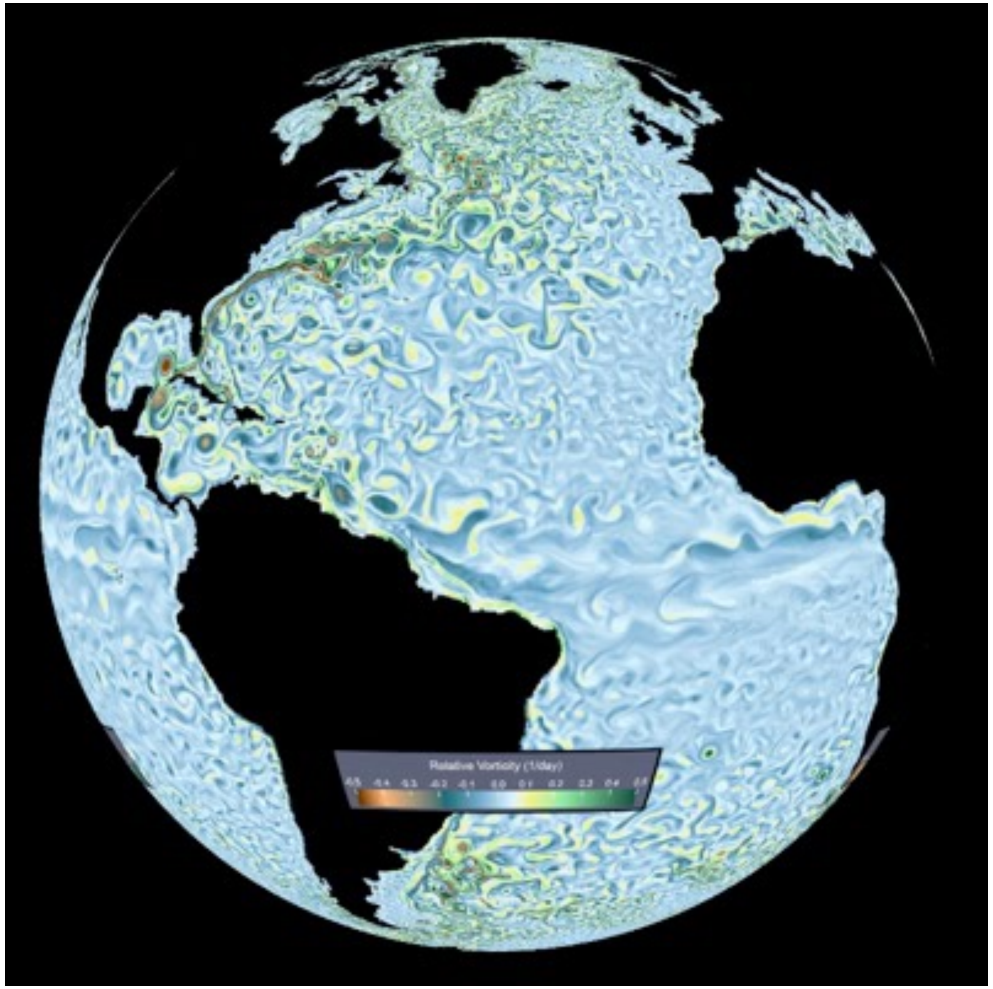
Return

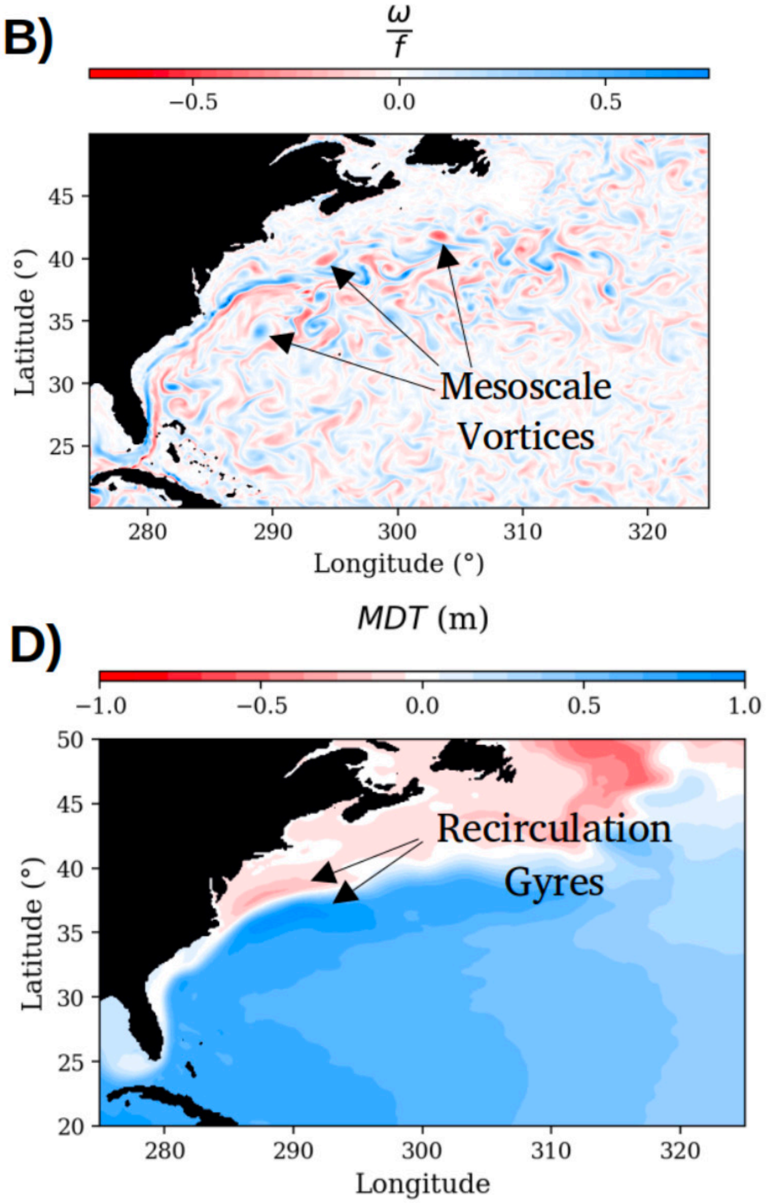
Stochasticity of turbulence

Stochasticity of turbulent flows



Stochasticity of turbulent flows



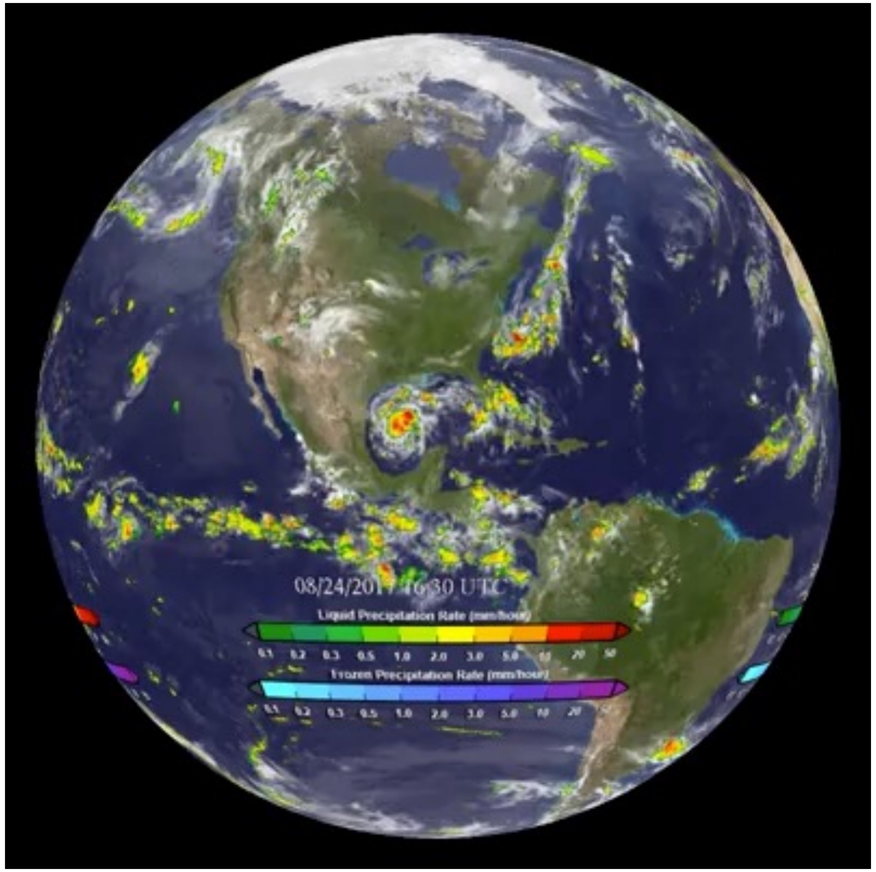


Random

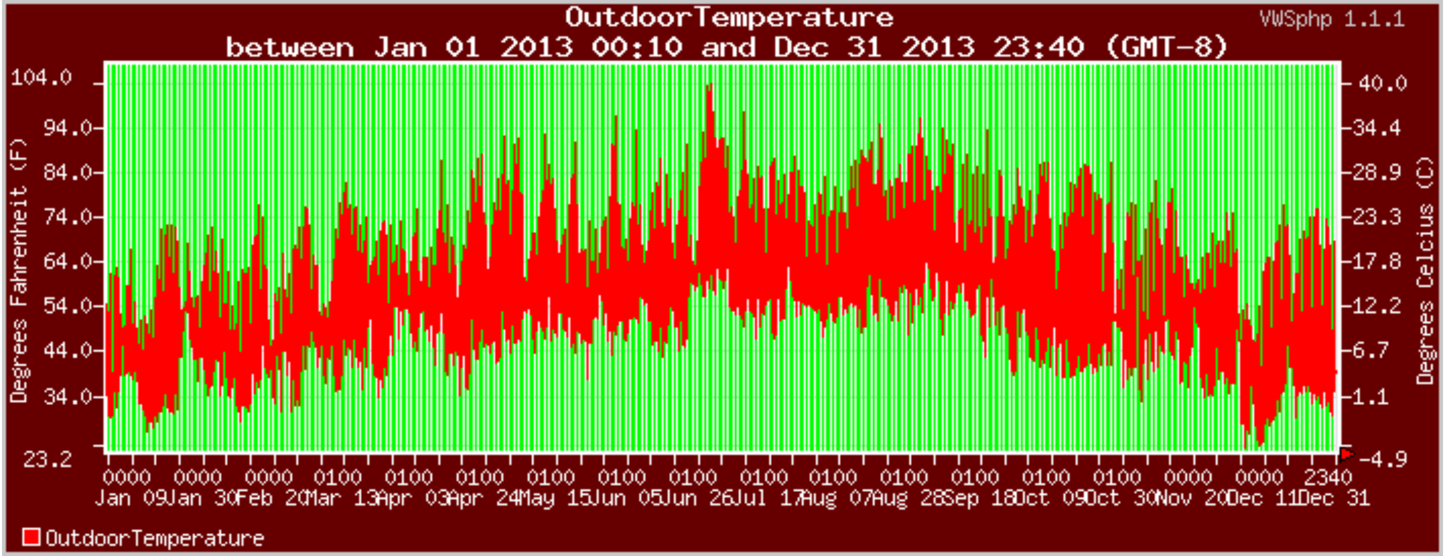
Mean

Miller PhD Thesis

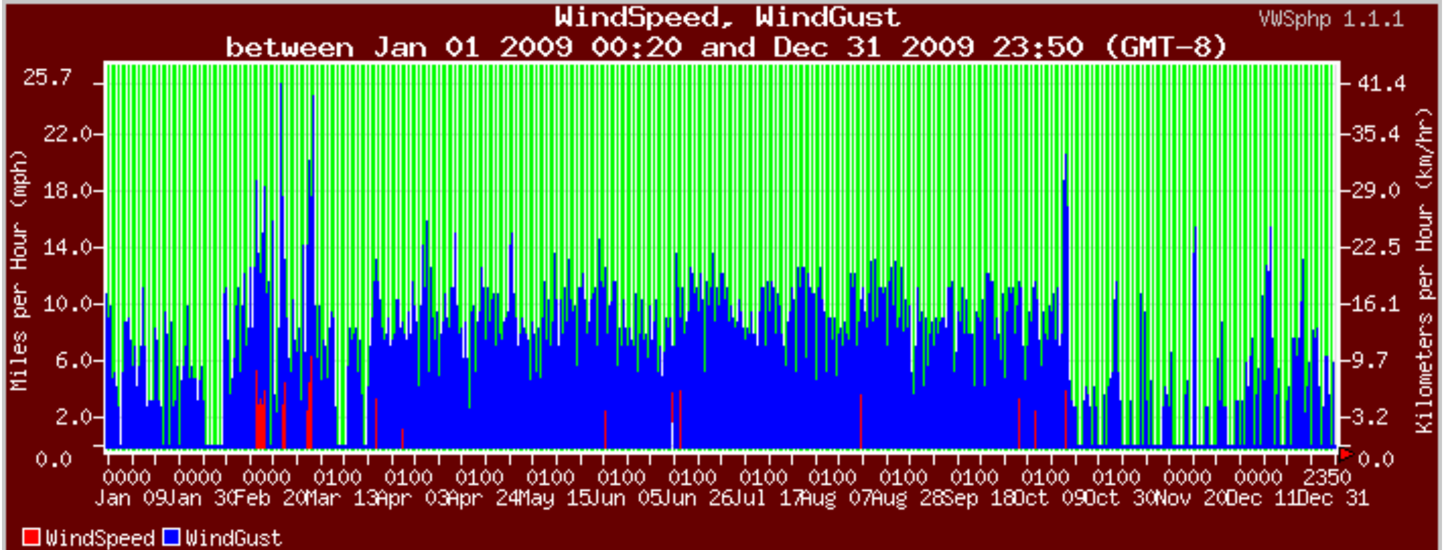
Stochasticity of turbulent flows



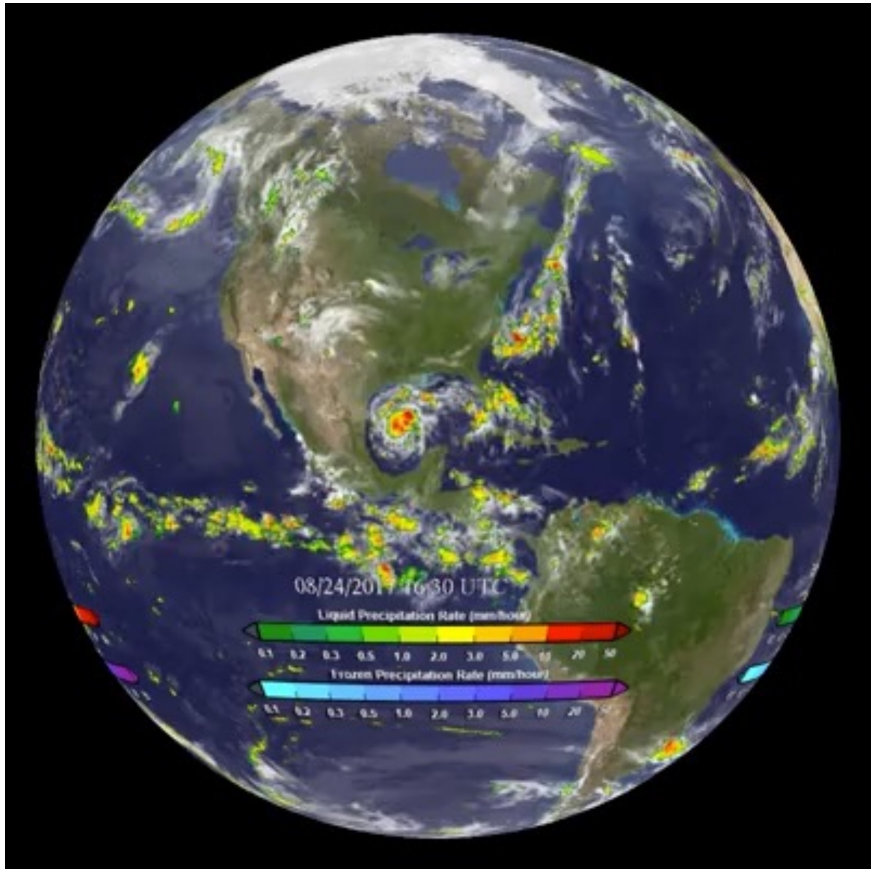
Temperature

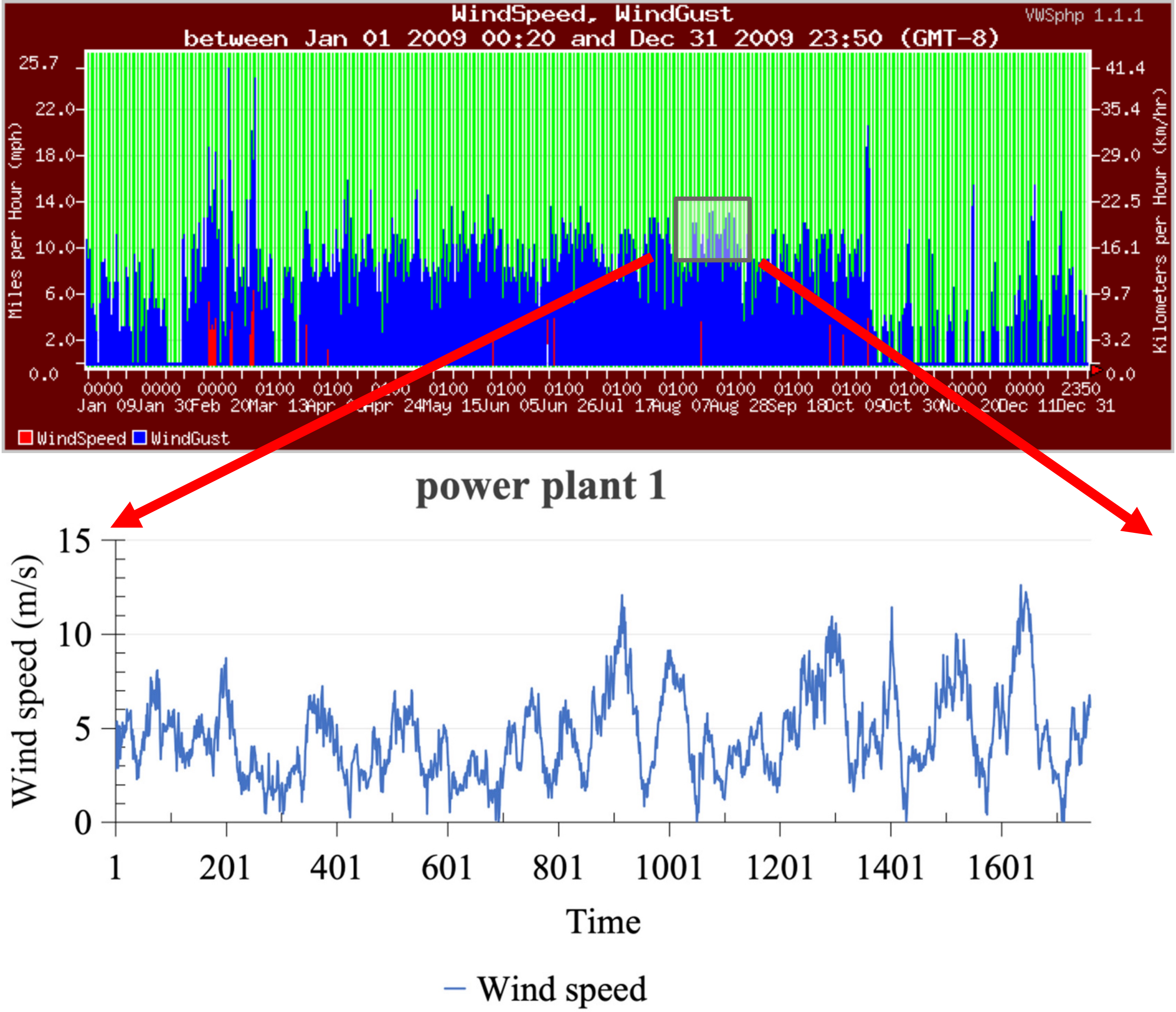


Wind

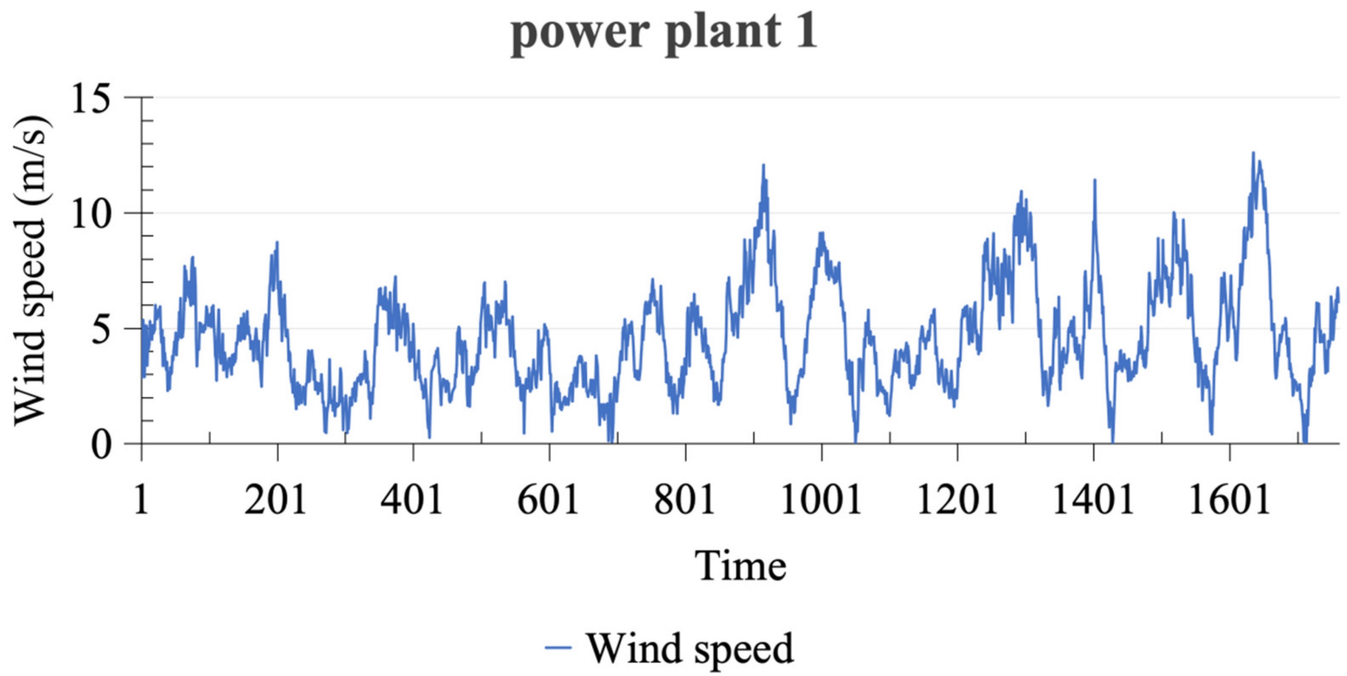


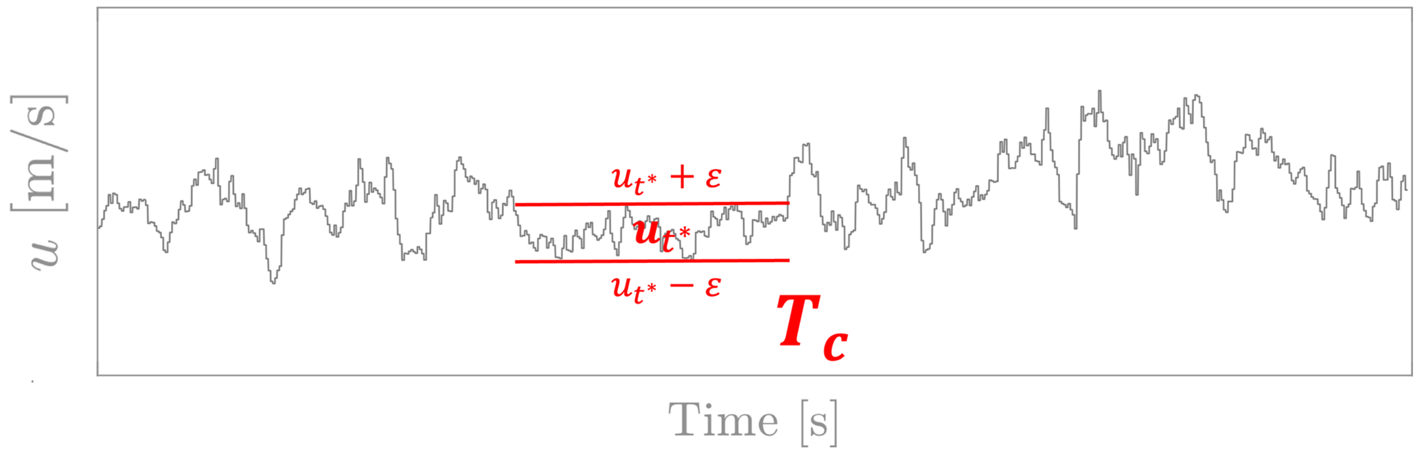
Stochasticity of turbulent flows





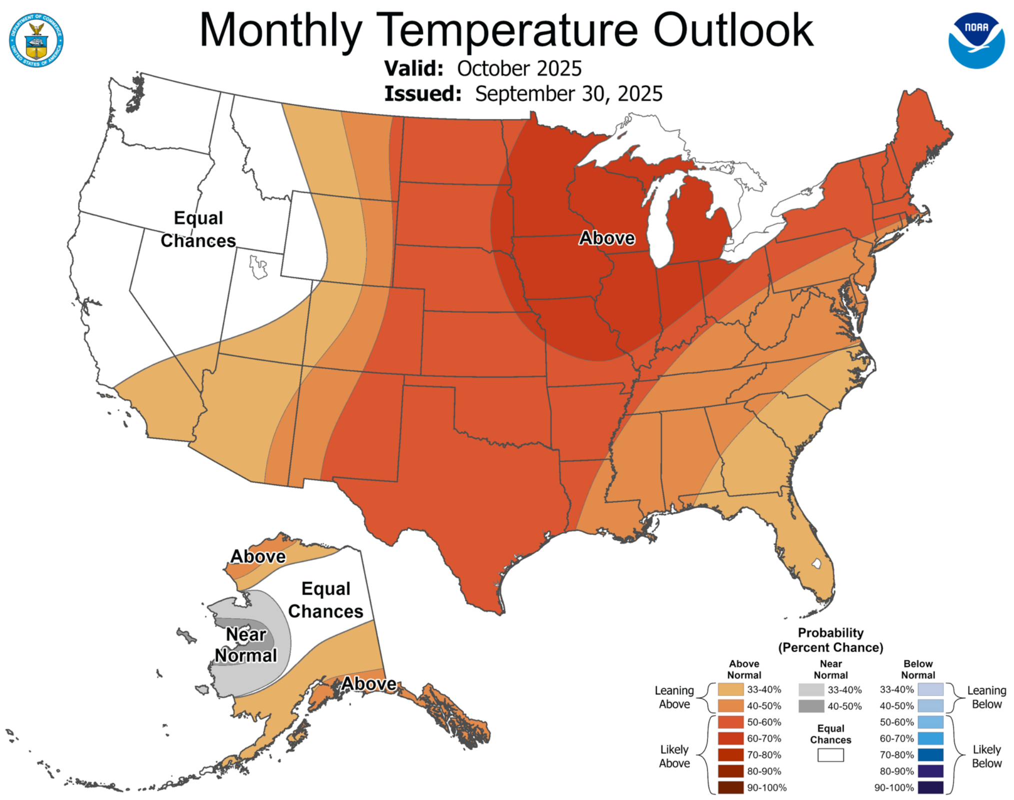
Stochasticity of turbulent flows

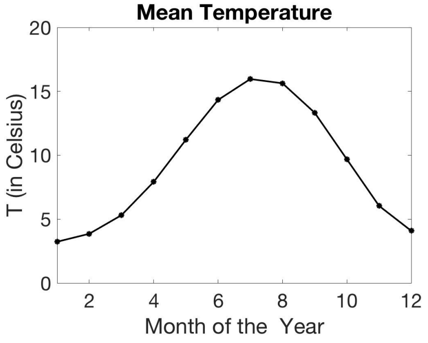


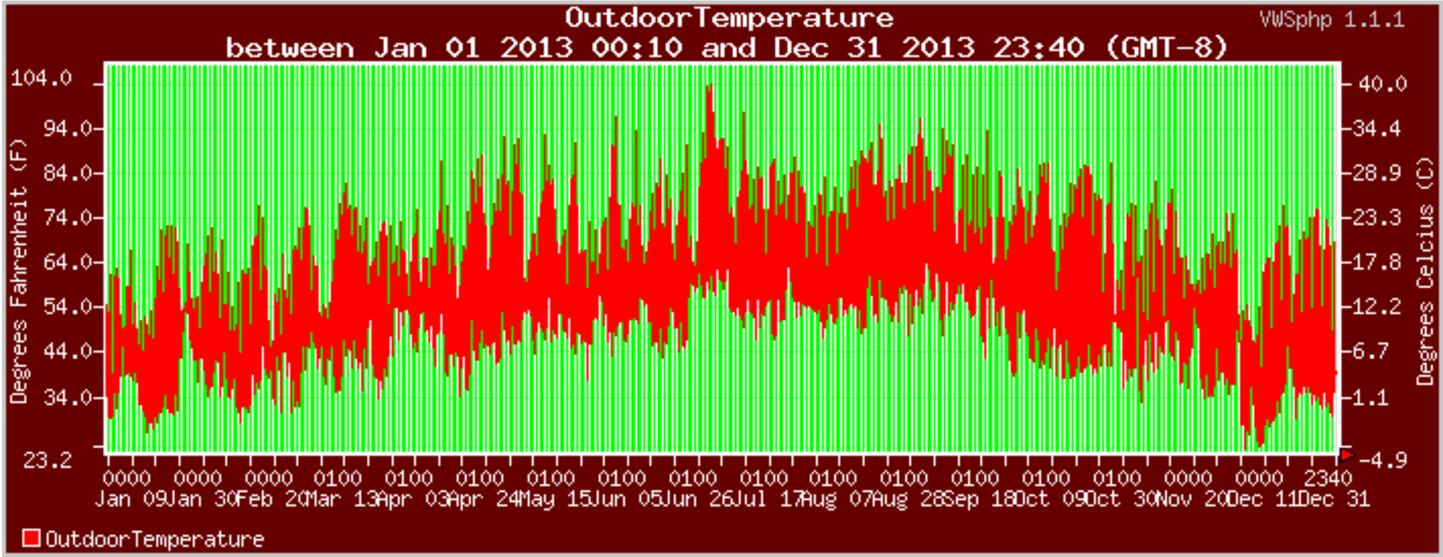


« Wind » defined as time averaged over 10mn

Stochasticity of turbulent flows







Monthly local average

« Climate »

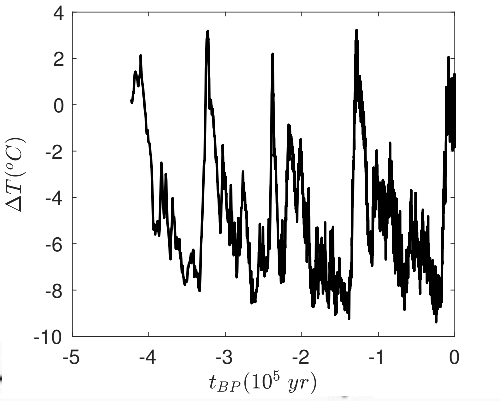
Daily local temperature

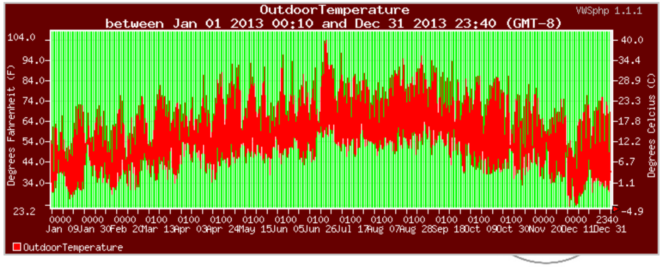
« Weather »

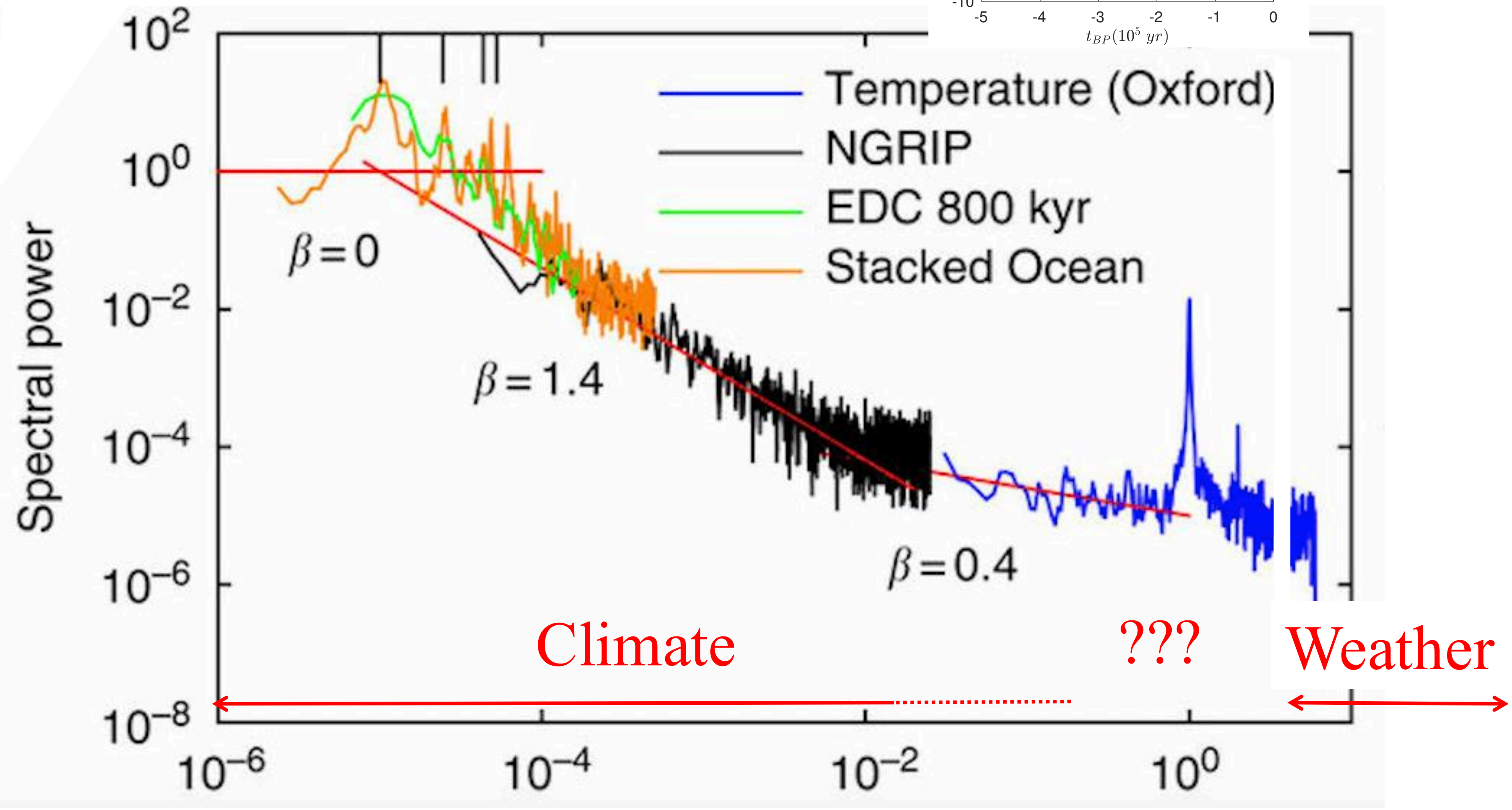
Back

Scale hierachy of turbulence

Scale hierachy

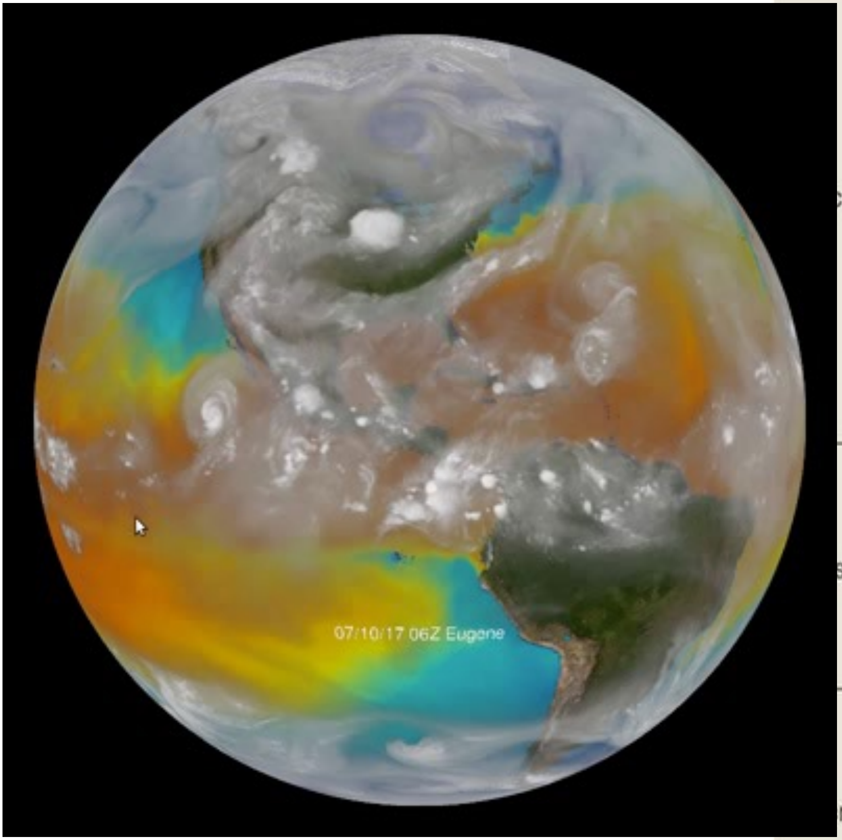


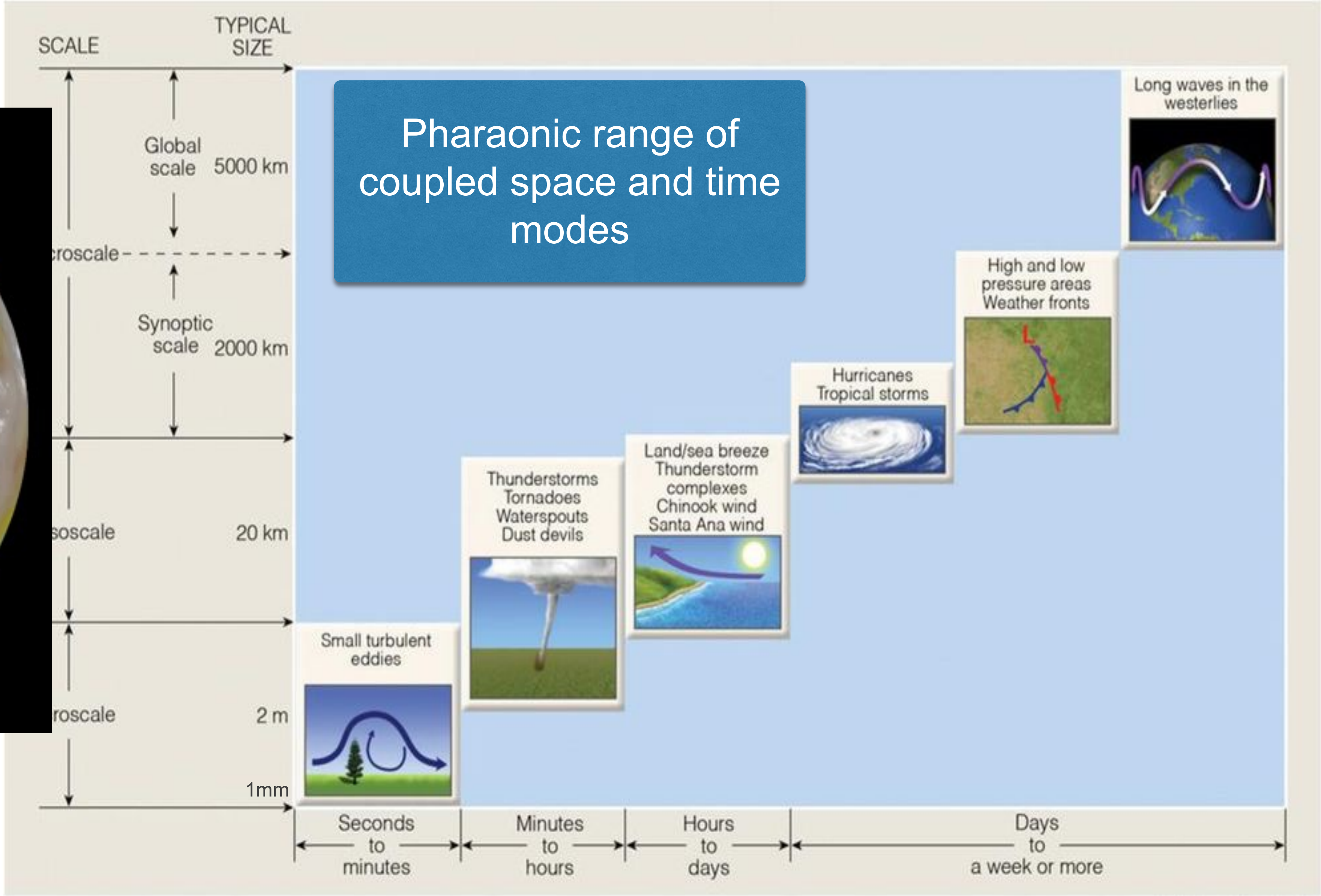




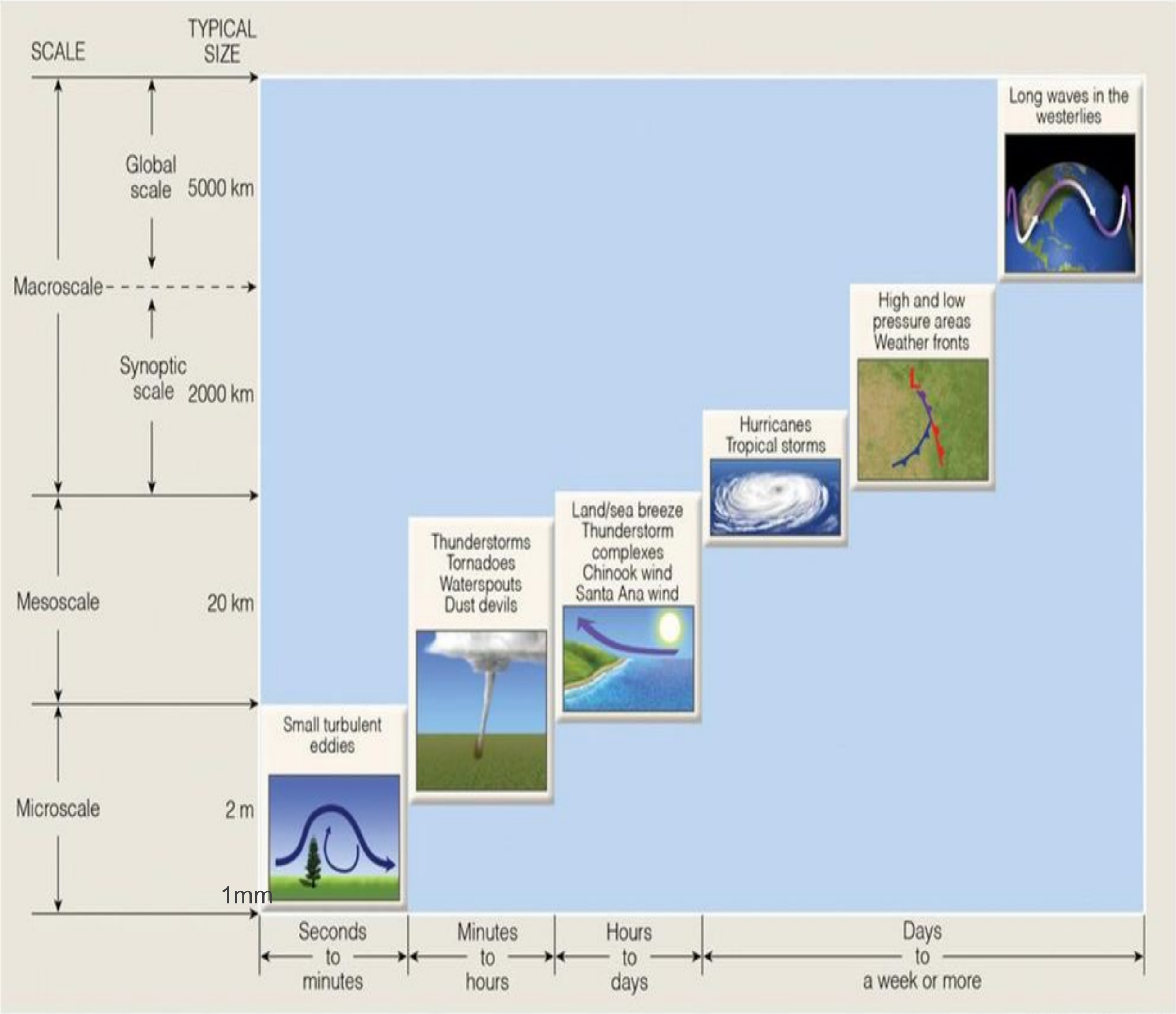
Scale hierachy

Pharaonic range of coupled space and time modes

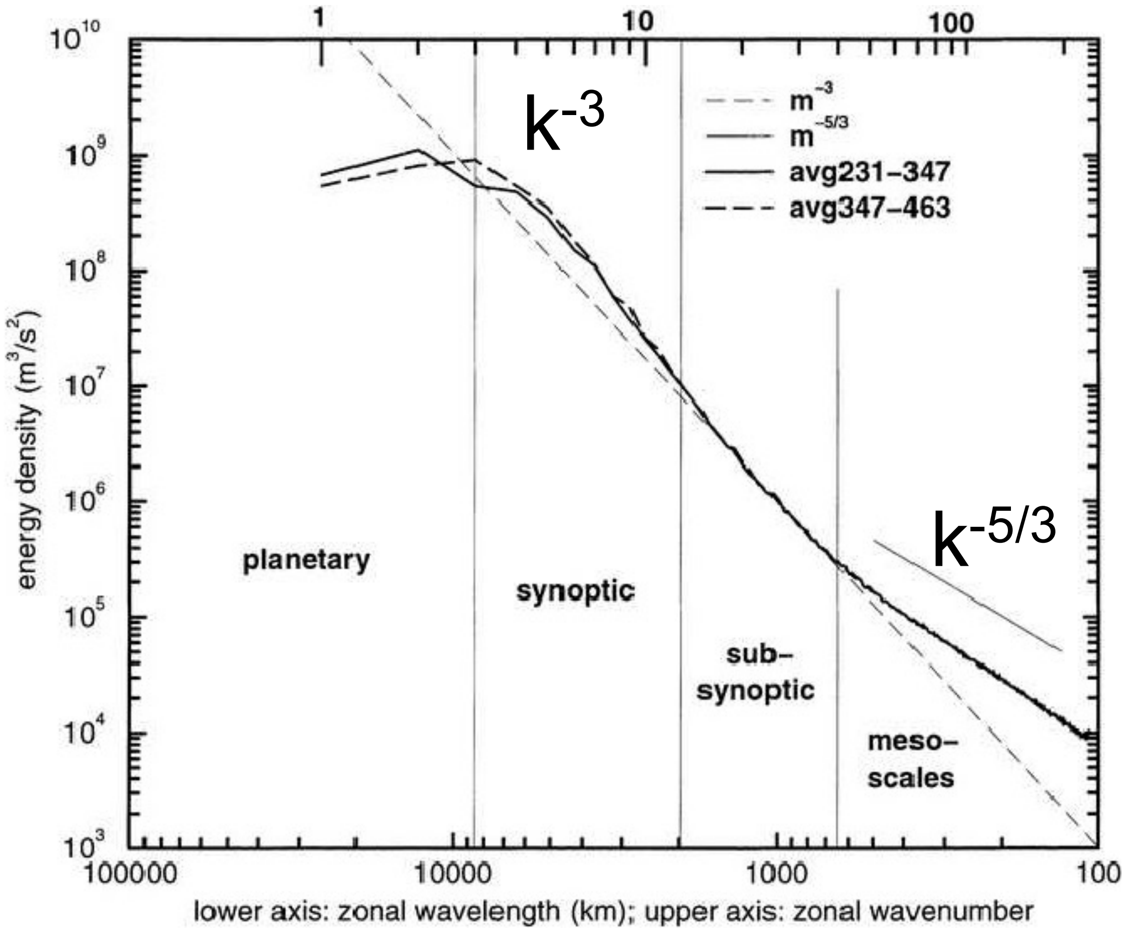




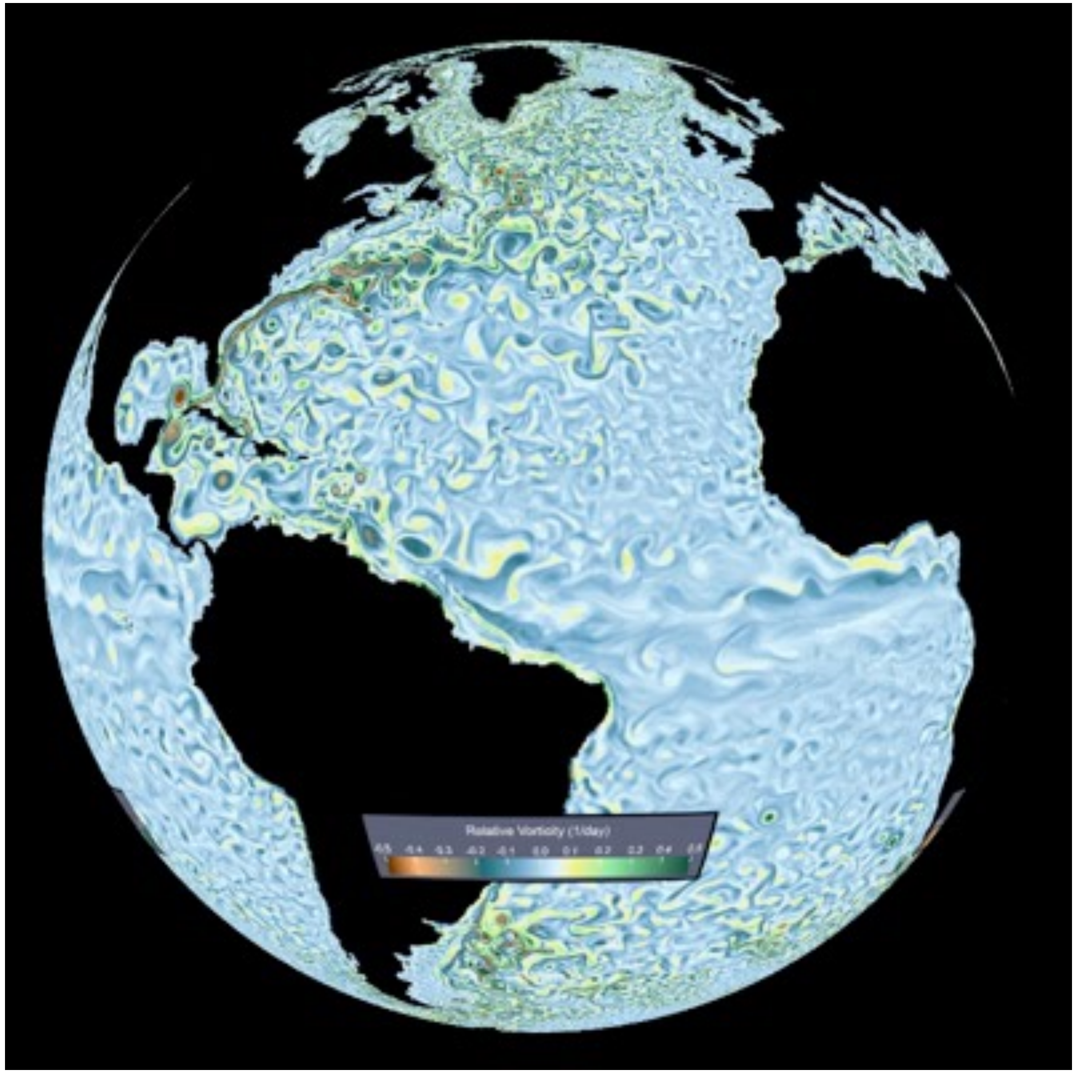
Scale hierachy



Nastrom-Gage spectrum



Scale hierarchy



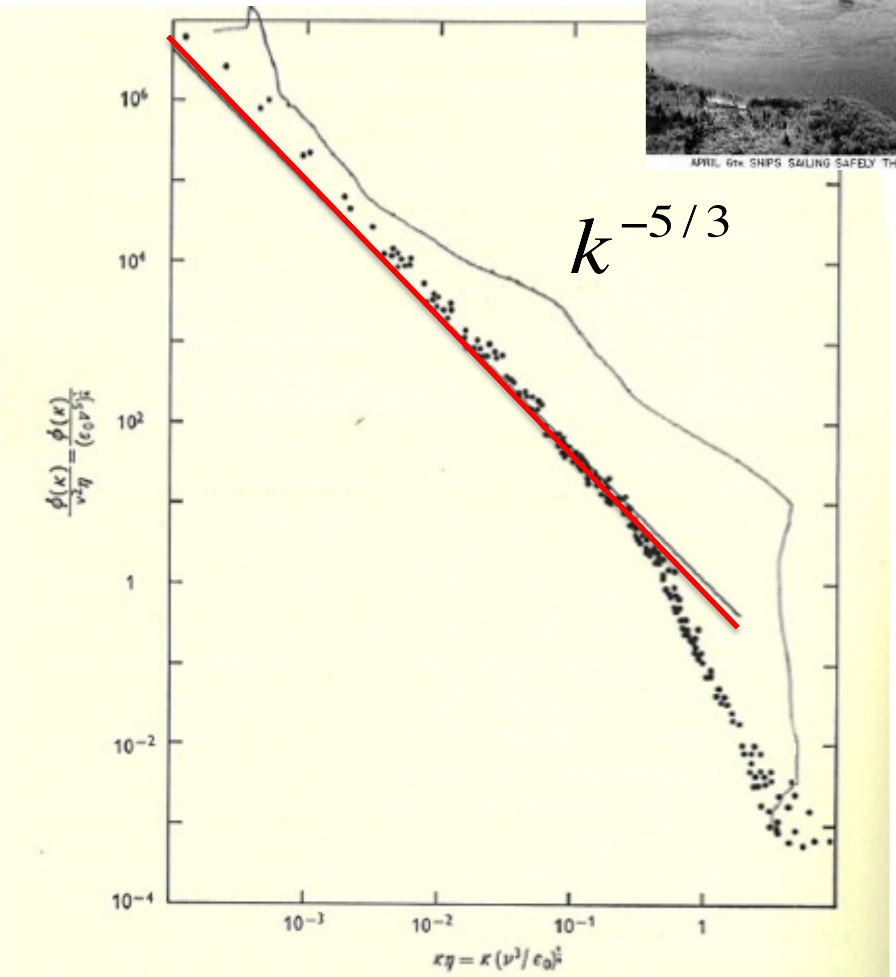


Fig. 6.2. The turbulence spectra, measured by Grant, Stewart and Moilliet (1962) and scaled according to the Kolmogorov parameters. The viscous dissipation rate ϵ_0 varied over a range of values of the order 100. The straight line represents variation as $\kappa^{-\frac{5}{3}}$. The top few points are believed to be rather high on account of the low frequency heaving motions of the ship.

Vortices are organized in a hierarchical way
They are regularized by viscosity

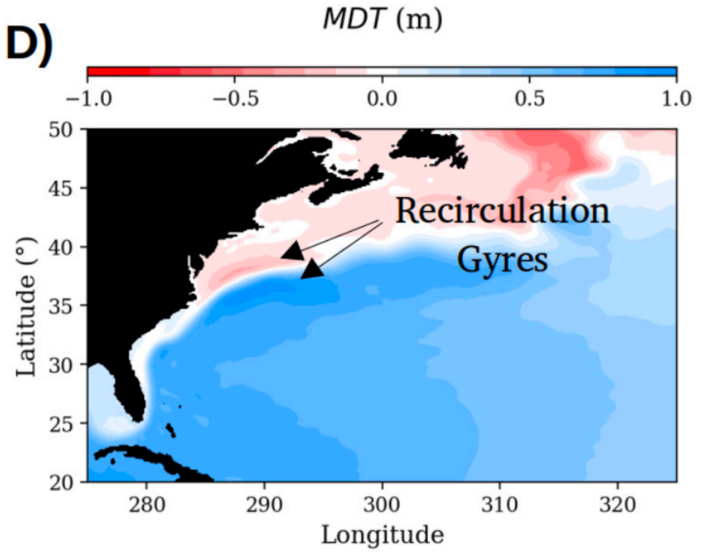
Back

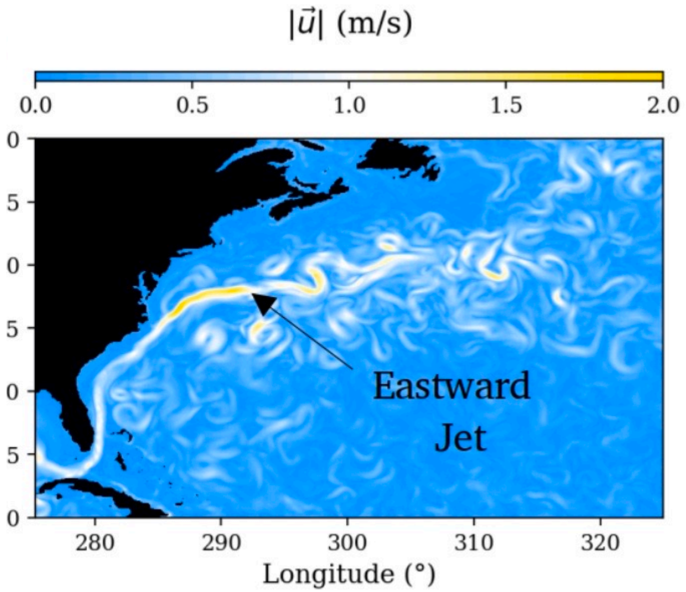
Intermittency of turbulence

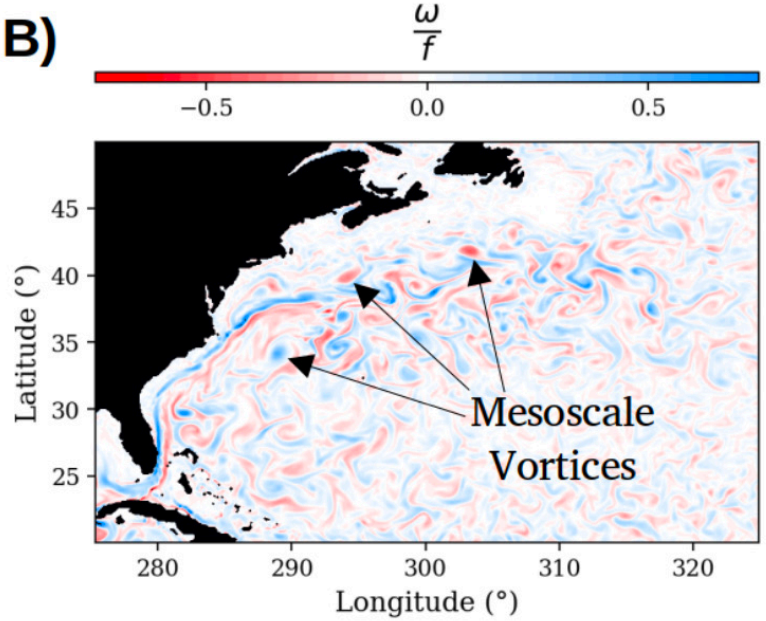
Space intermittency: Breaking of the space translation symmetry

Mean stream function | Velocity | Vorticity

Large scale | Intermediate scale | Small scale



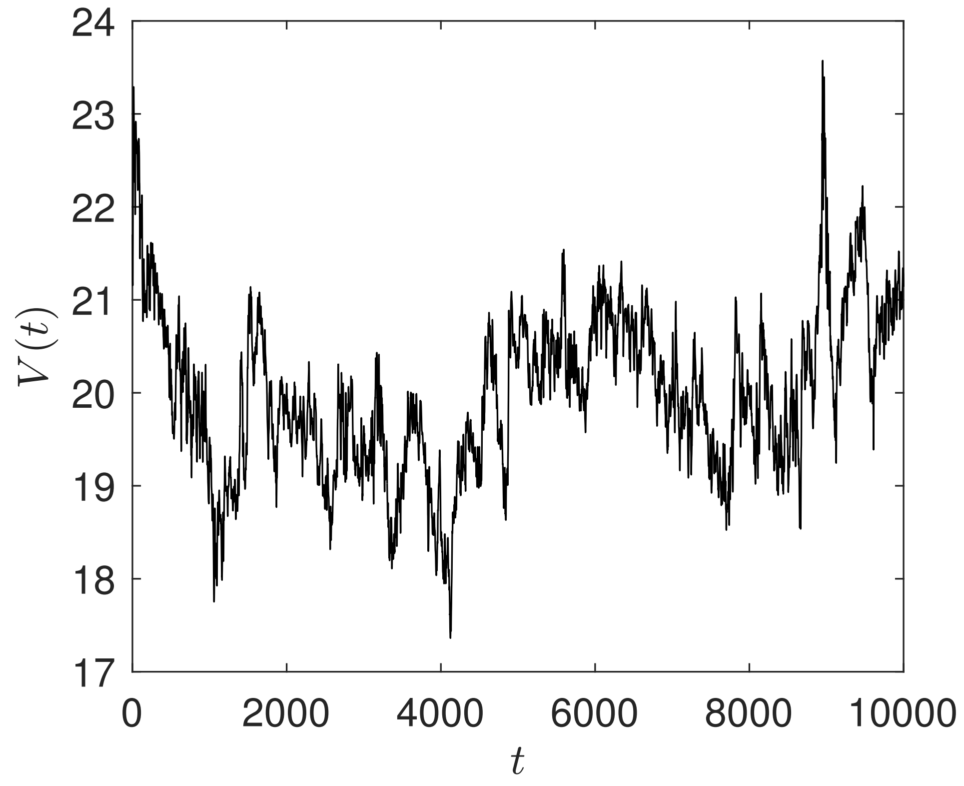




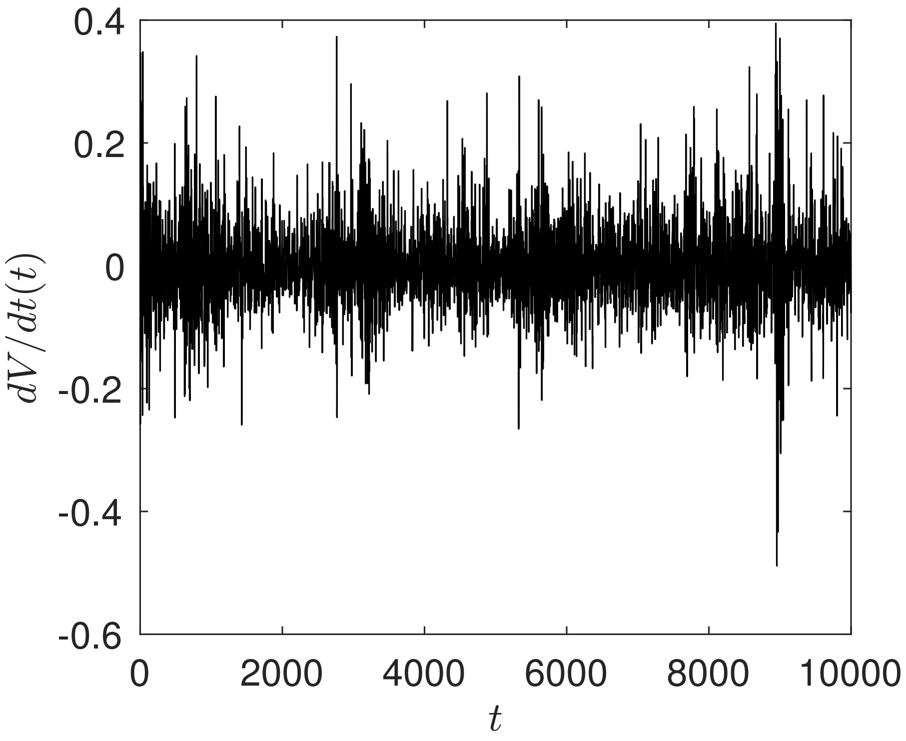
Homogeneous | Less homogeneous | Inhomogeneous

Miller PhD thesis

Time Intermittency: breaking of the time translation

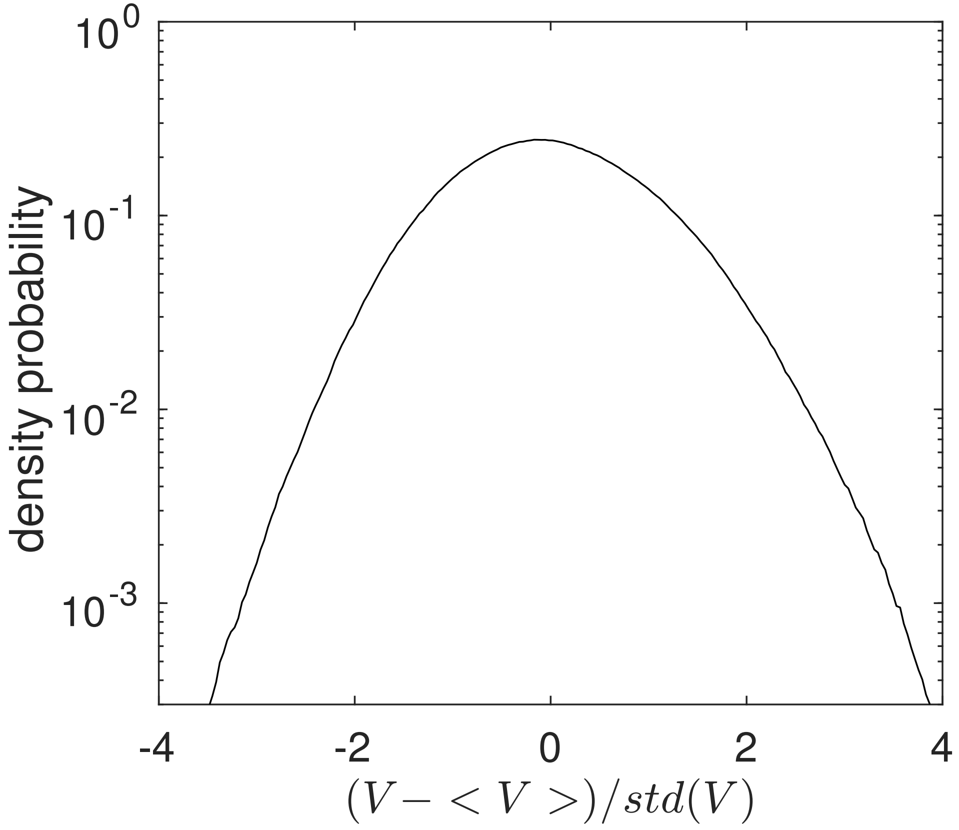


Velocity

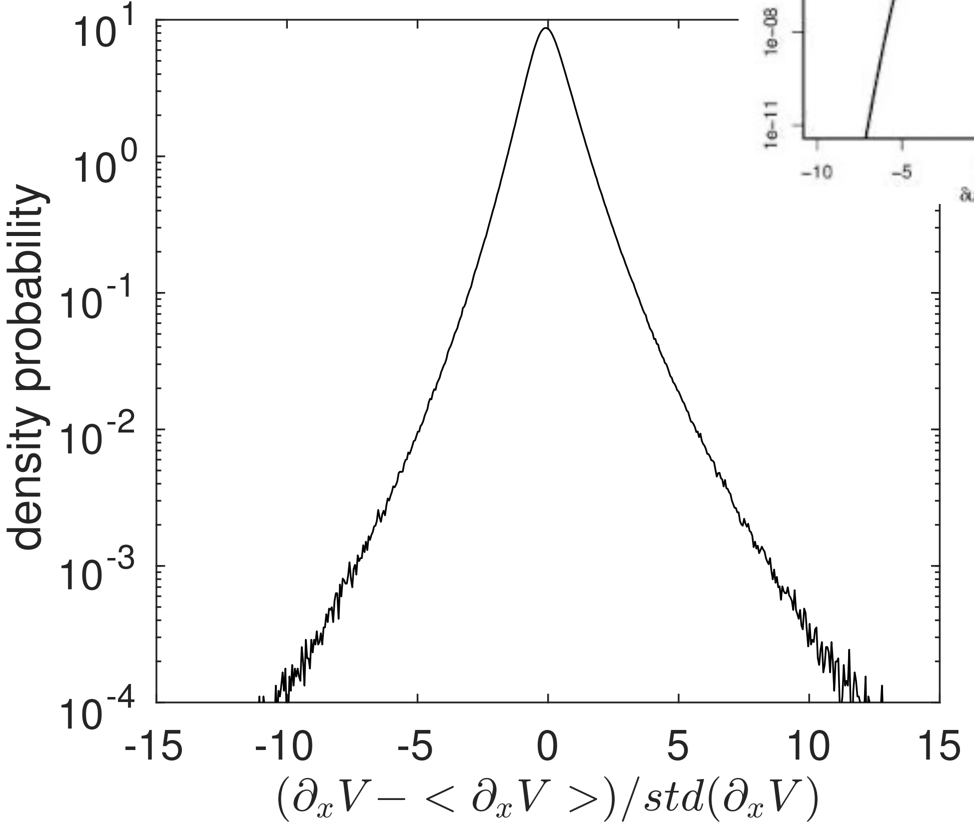


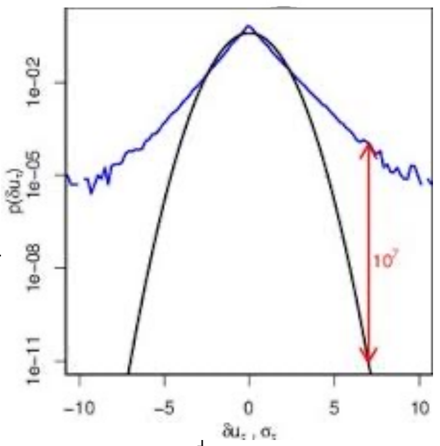
Derivative

Time Intermittency: breaking of the time translation



Velocity





Derivative

Back

The transport regimes of turbulence

Sailing with the Ducks

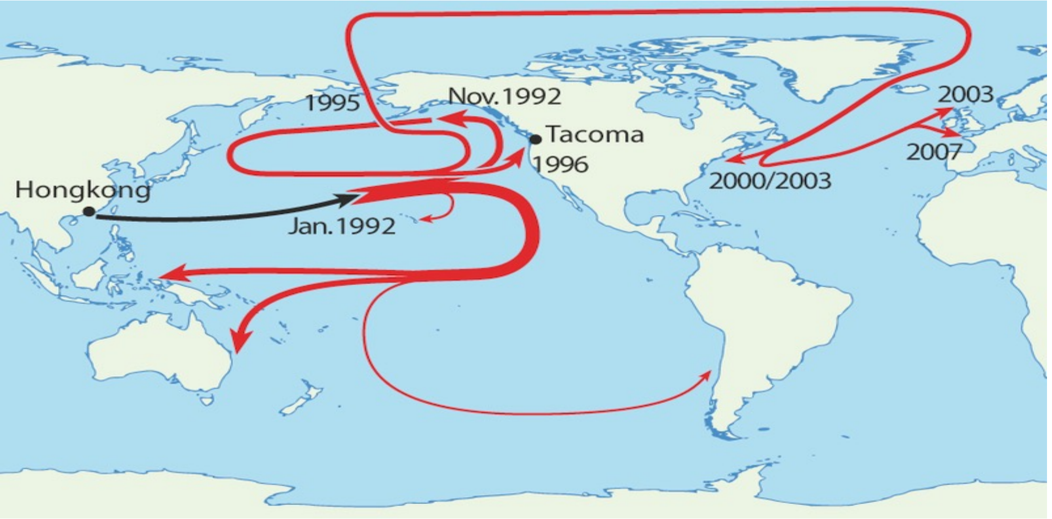
TRACERS OF TURBULENT FLOW

In 1992 a shipping crate bound for the US from Hong Kong fell overboard and was lost at sea. The cargo that spilled into the ocean was 28,000 plastic duck bath toys.

Those toys have been following ocean currents ever since reaching every continent but Antarctica, including surviving the Arctic Ocean and moving around North America to reach the British Isles.

Scientists have been using the data collected from ships spotting the yellow tide or duckies washing ashore to study ocean patterns and how long it takes for them to navigate the globe

https://en.wikipedia.org/wiki/Friendly_Floatees_spill



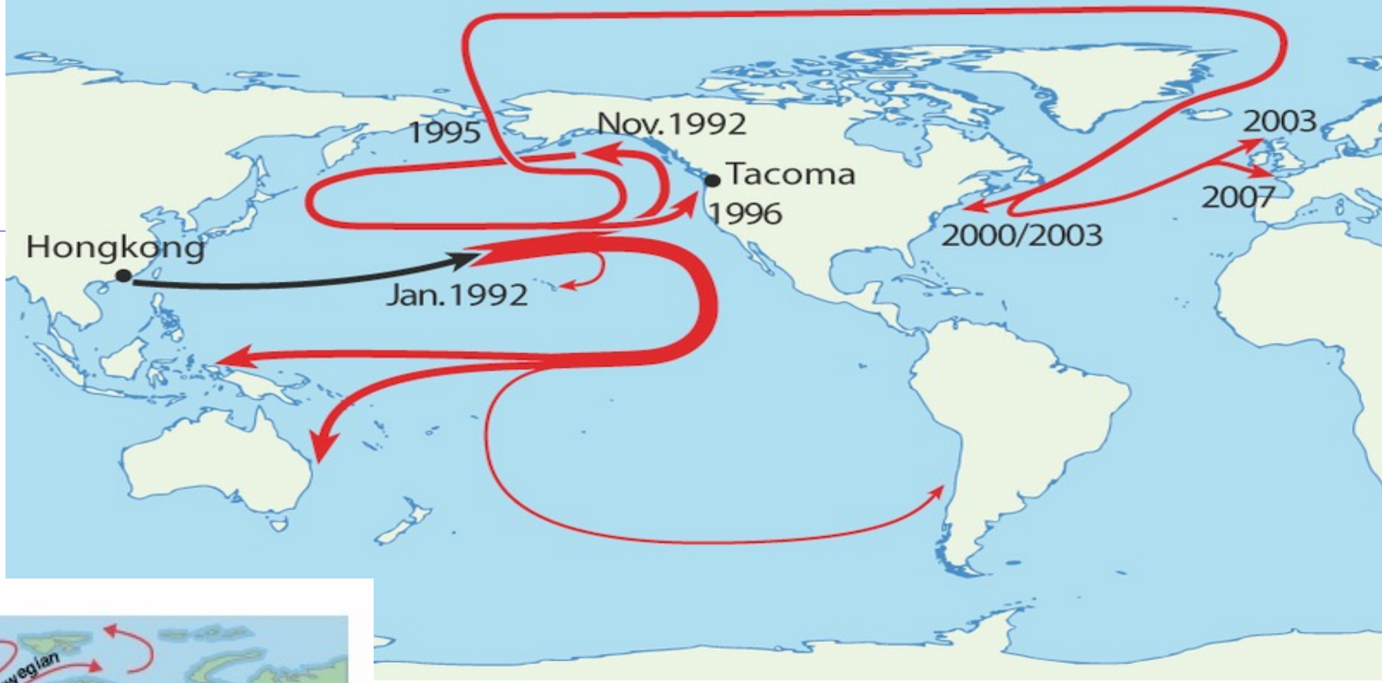
The plastic vortex

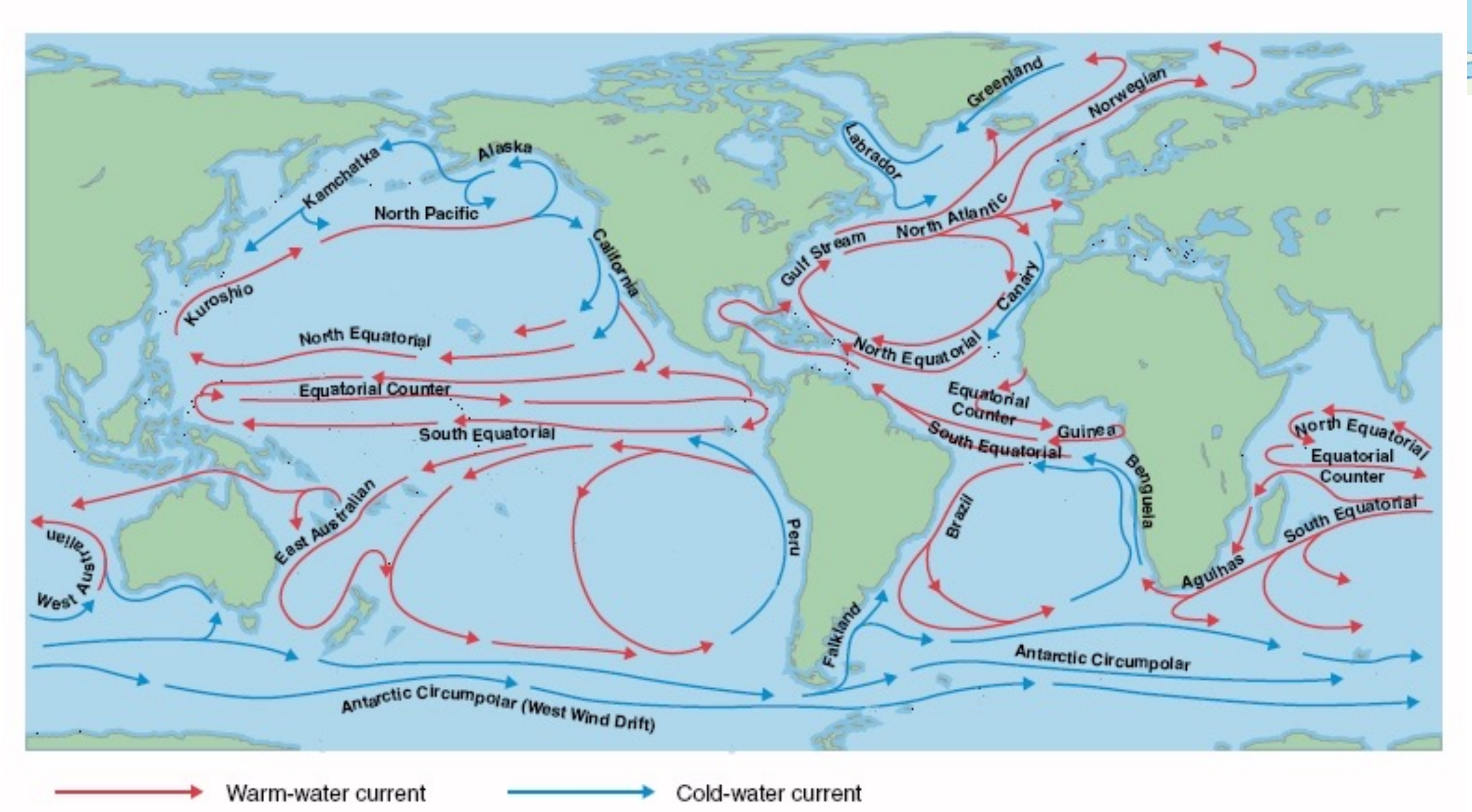
CAPTURE BY TURBULENCE

Sailing with the Ducks

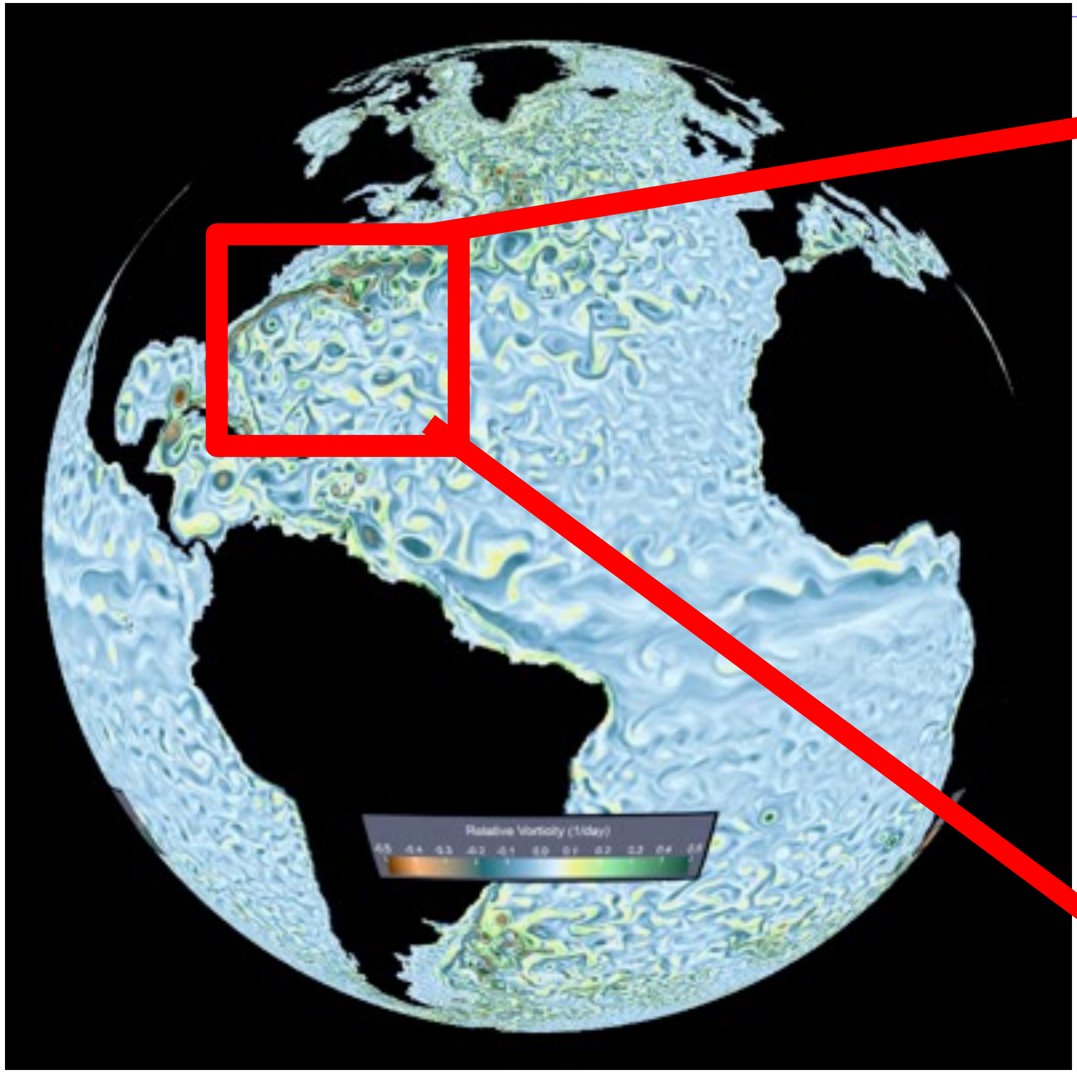
TRANSPORT BY TURBULENCE

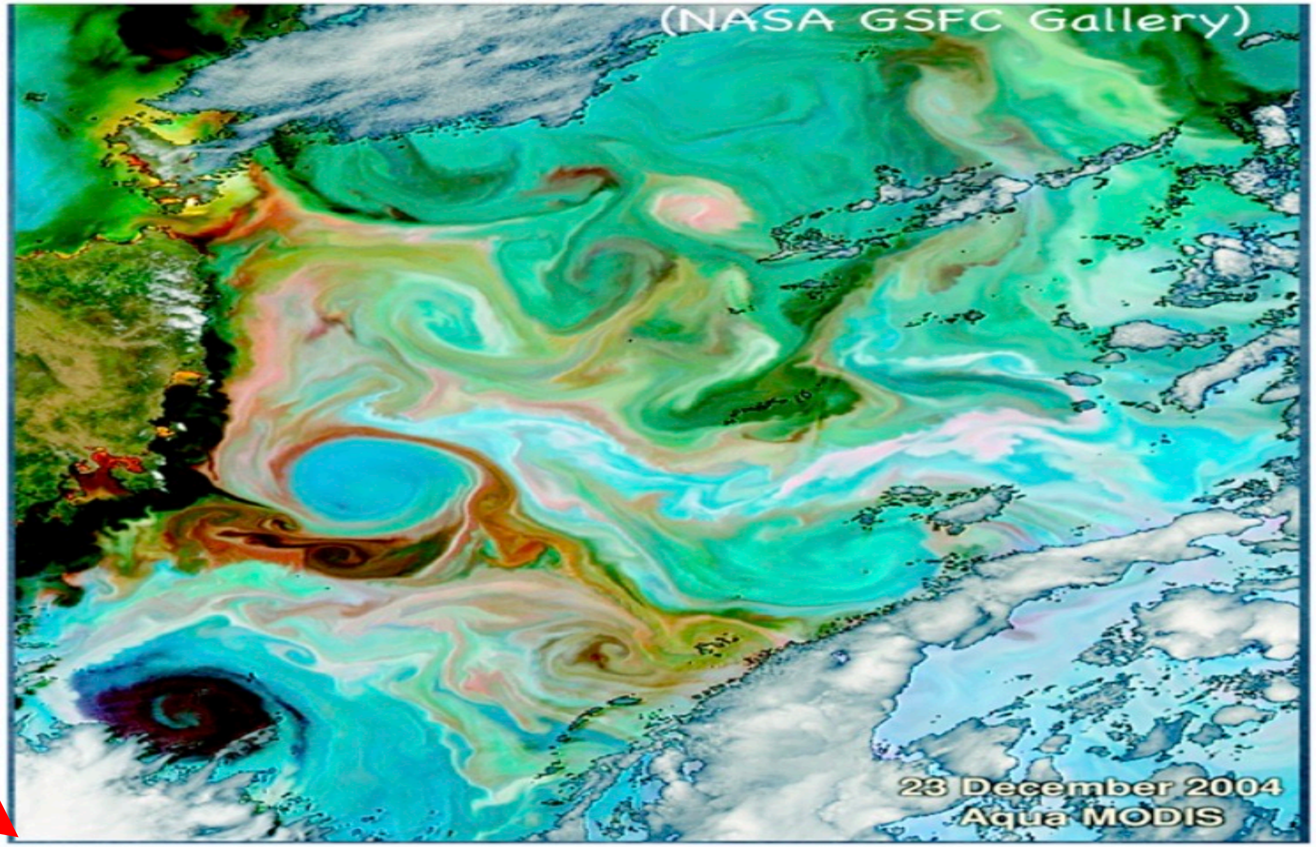
Ducks are transported by current



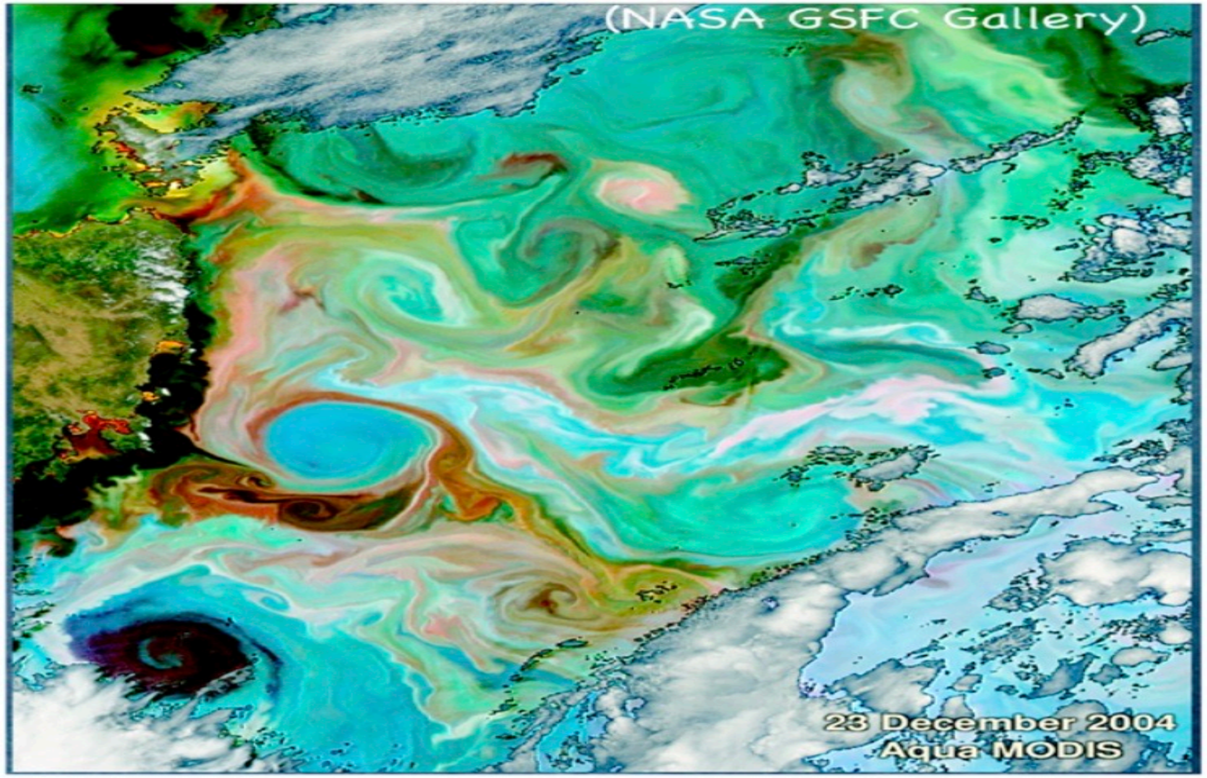


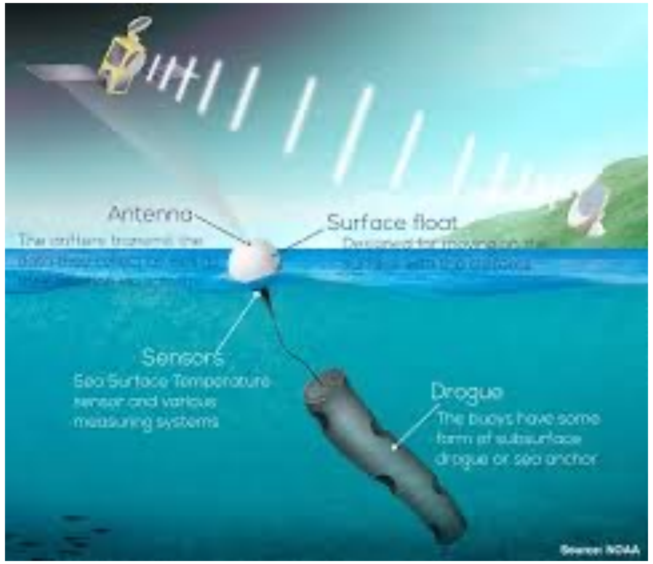
Drifter in oceanic vortices



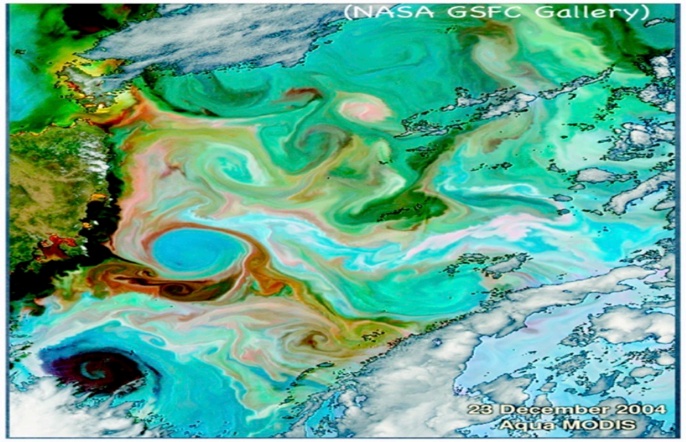


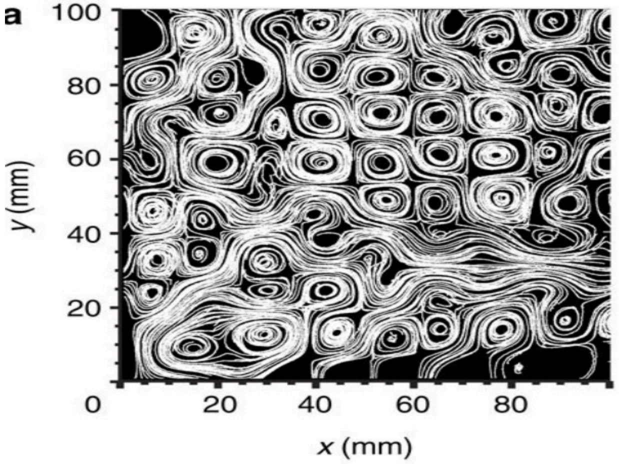
Drifter in oceanic vortices

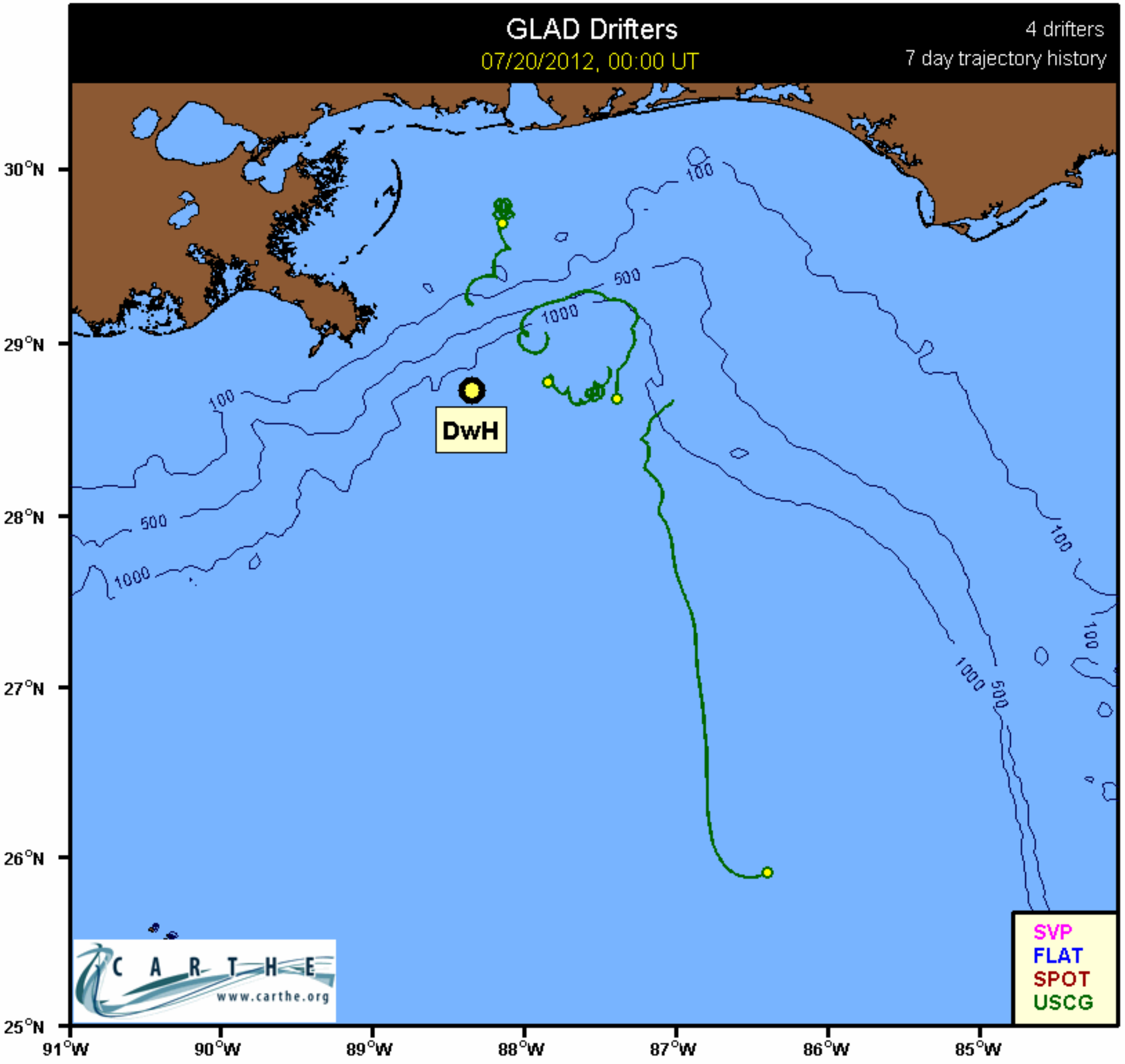




Drifter in oceanic vortices

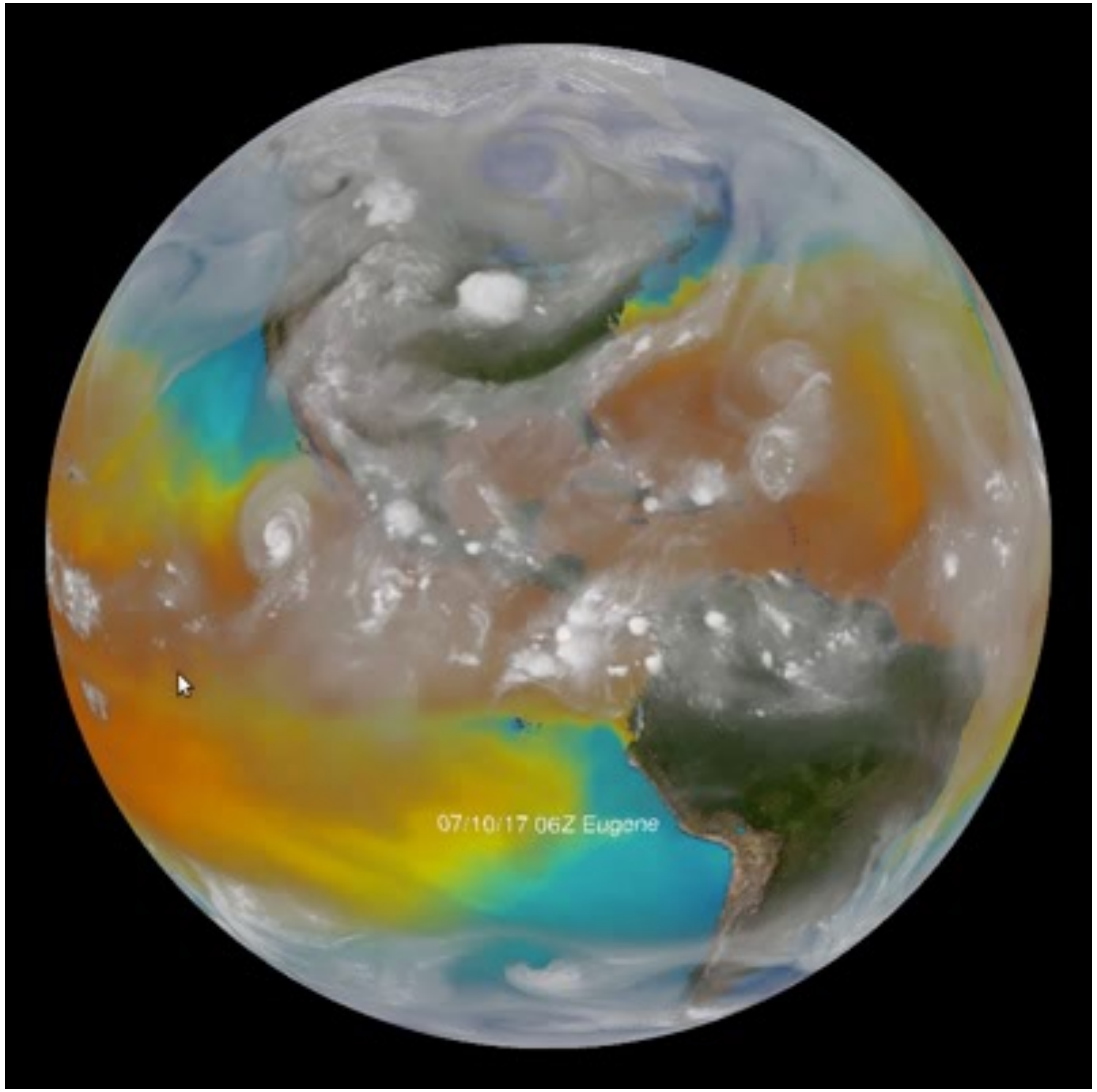


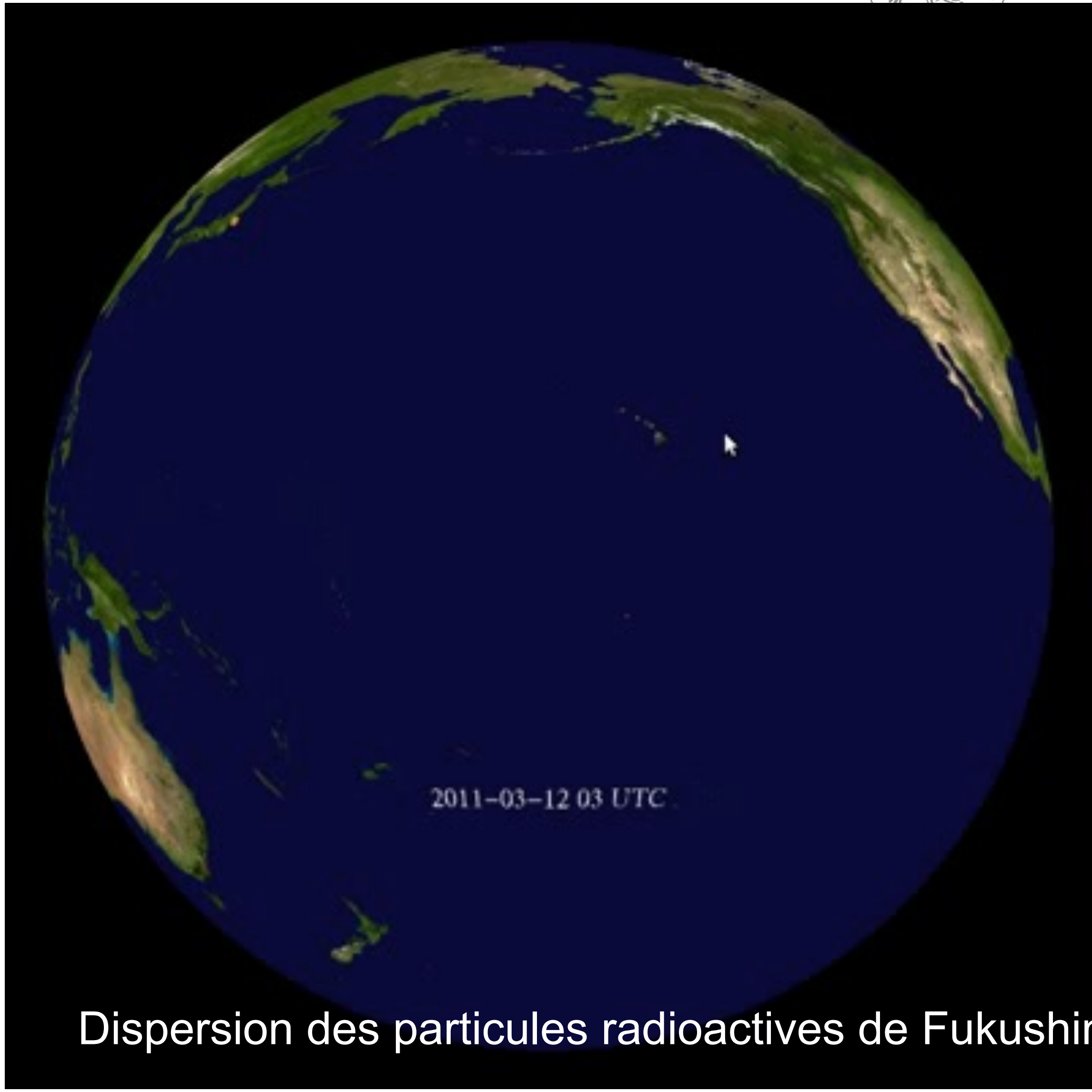




Drifters are transported and ejected

Dispersion of particles in atmosphere





Particle dispersion: one point statistics

$$< \Delta x^2 >$$

How does a cloud of particle evolve depending on the fluid condition?

Particle dispersion: two point statistics

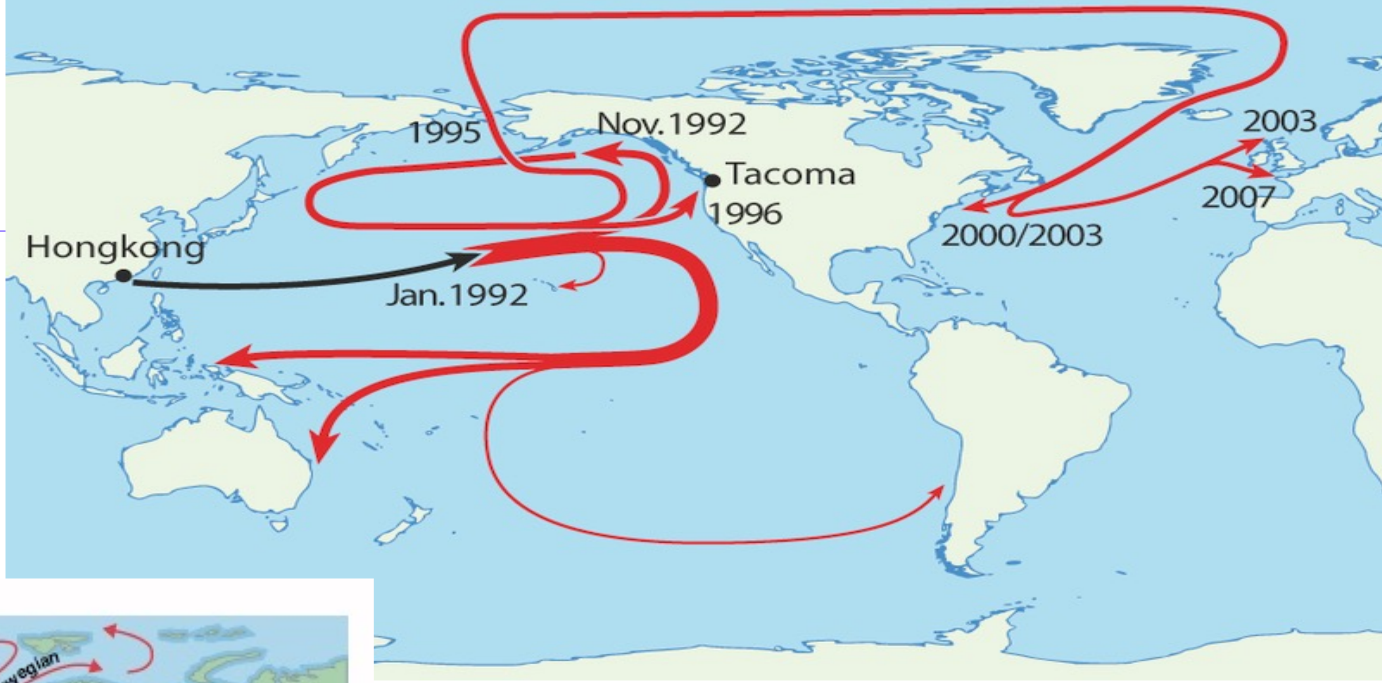
$$< \delta R^2 >$$

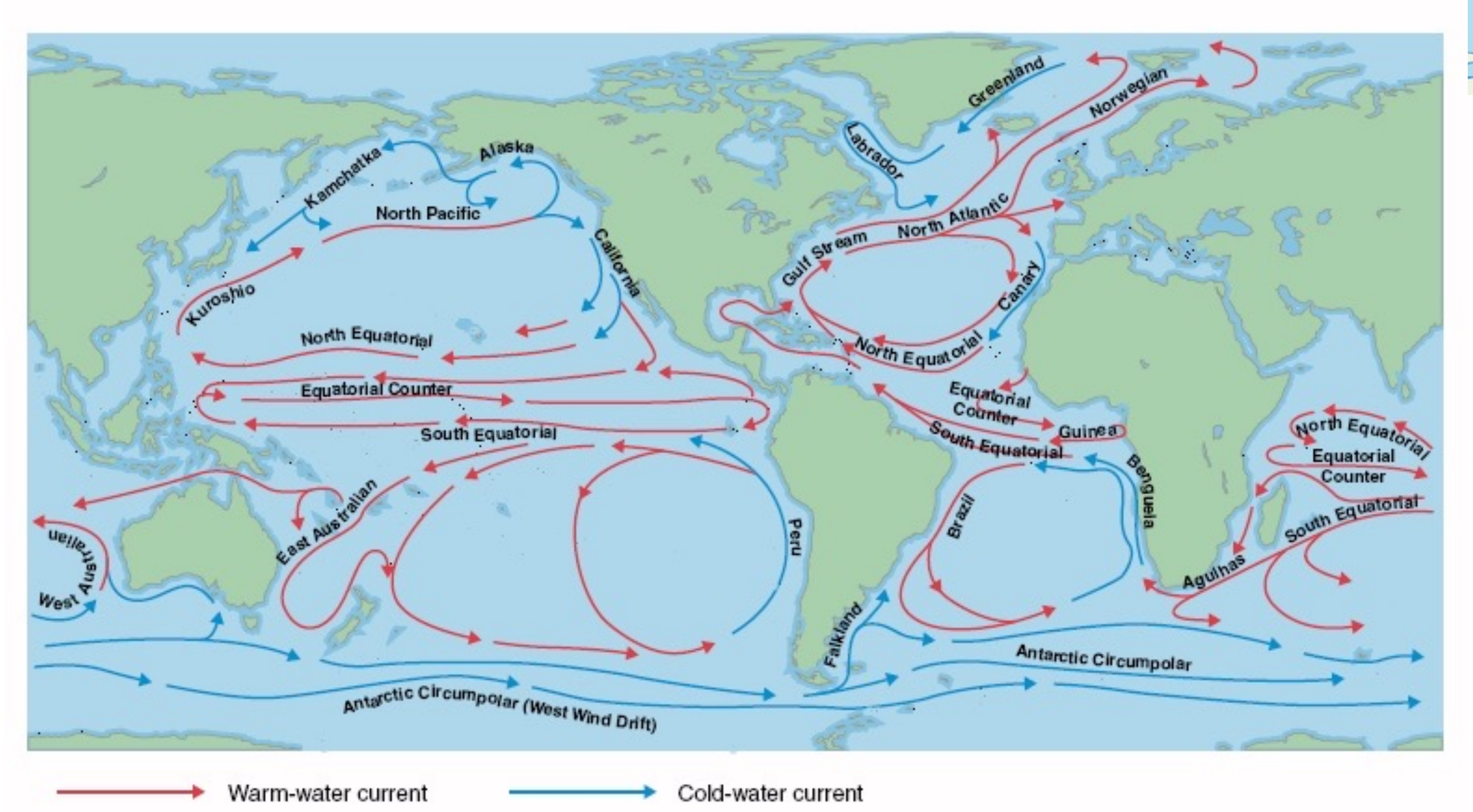
How does a cloud of particle evolve depending on the fluid condition?

Sailing with the Ducks

TRANSPORT BY TURBULENCE

Ducks are transported by current



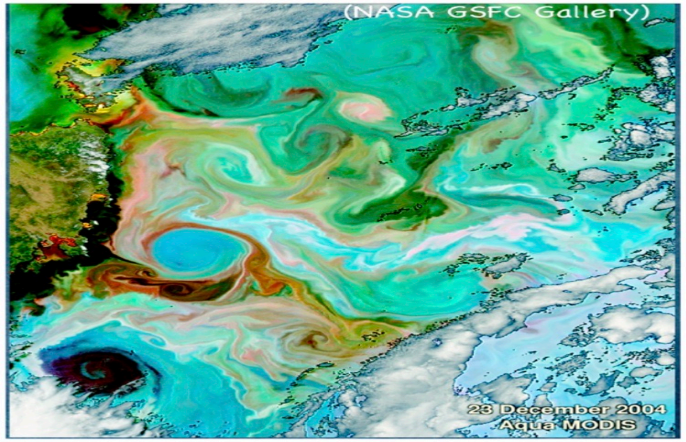


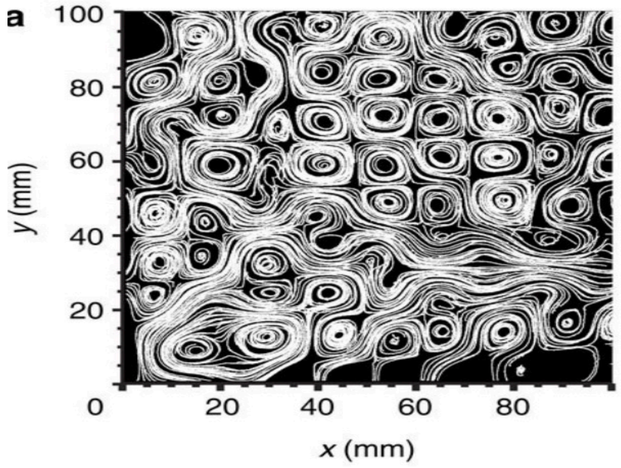
Ballistic motion

$$< \Delta x^2 > \sim (\sigma_u t)^2$$

$$< \delta R^2 > \sim R_0^{2/3} t^2$$

Drifter in oceanic vortices: chaotic motion



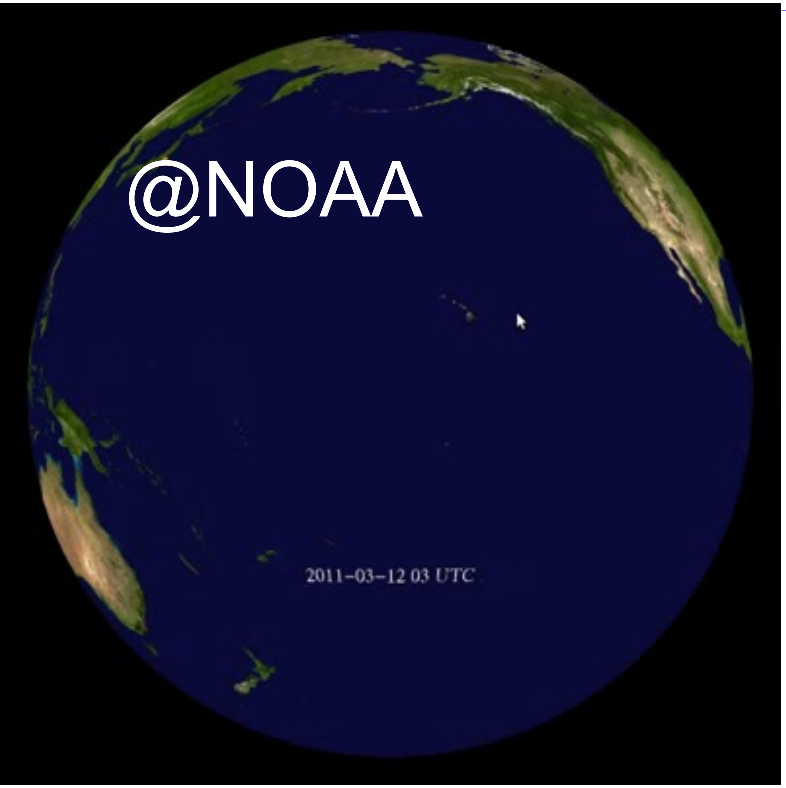


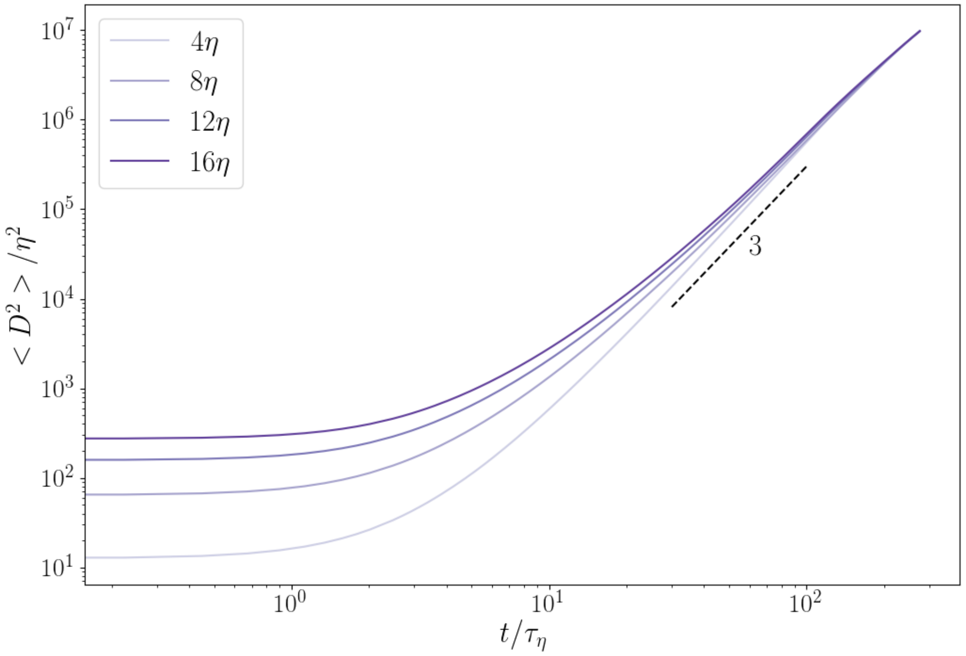
$$< \delta R^2 > \sim R_0^2 e^{\Lambda t}$$

Memory of Initial Dispersion

Drifters are transported and ejected

Dispersion of particles in atmosphere: Richardson law





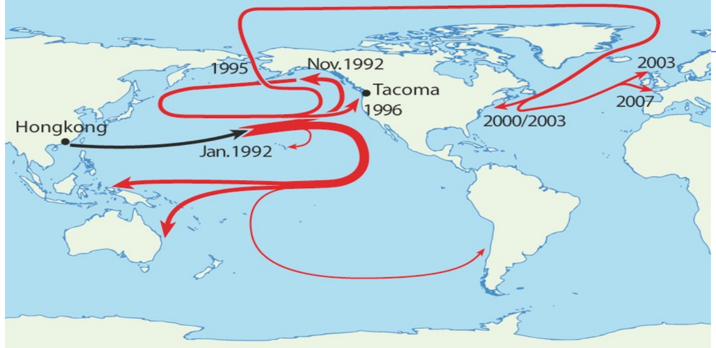
$$< \Delta x^2 > \sim \sigma_u^2 t$$

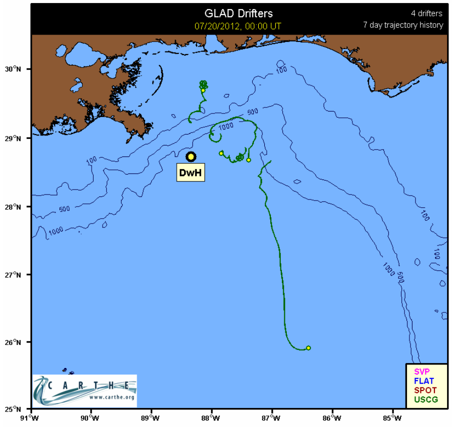
$$< \delta R^2 > \sim t^3$$

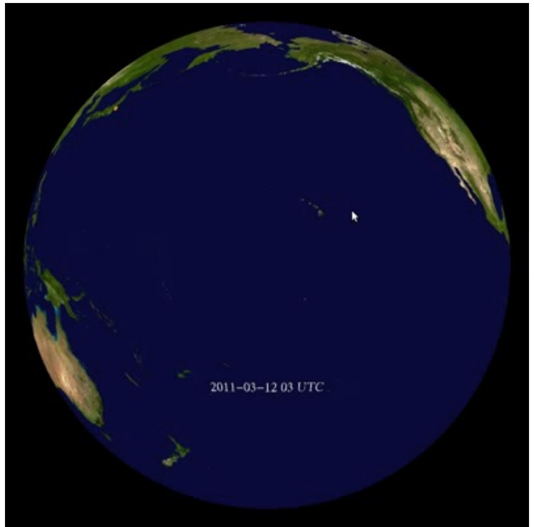
Independent of initial condition

Why turbulence is usefull (or not?)

BALLISTIC/CHAOTIC VS TURBULENT MIXING





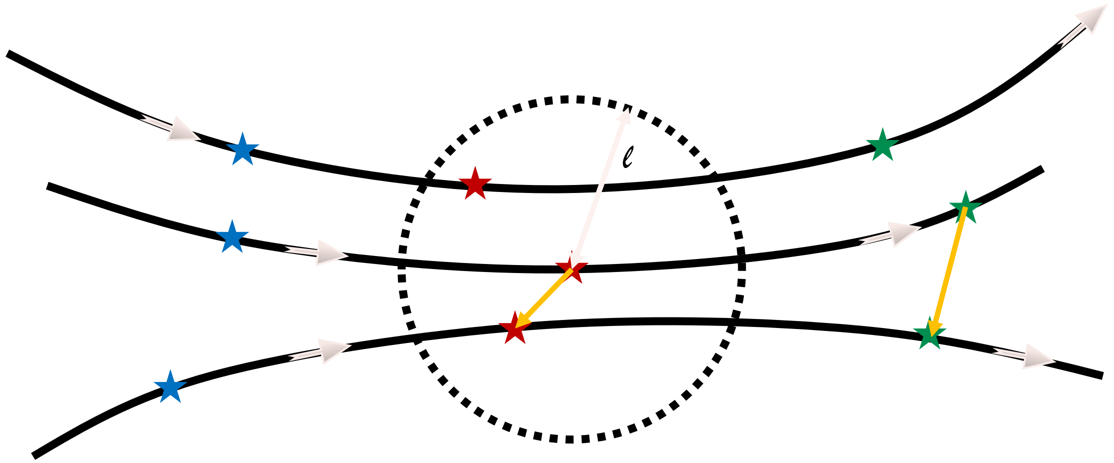


	Theoretical Diffusive time	Observed mixing time
Water 10^4 km	10^{13} years	9 month
Water 10^3 km	10^{11} years	A few days
Air 10^4 km	10^{12} years	A few days

Why turbulence is usefull

CHAOTIC VS TURBULENT MIXING

Irreversibility of dispersion



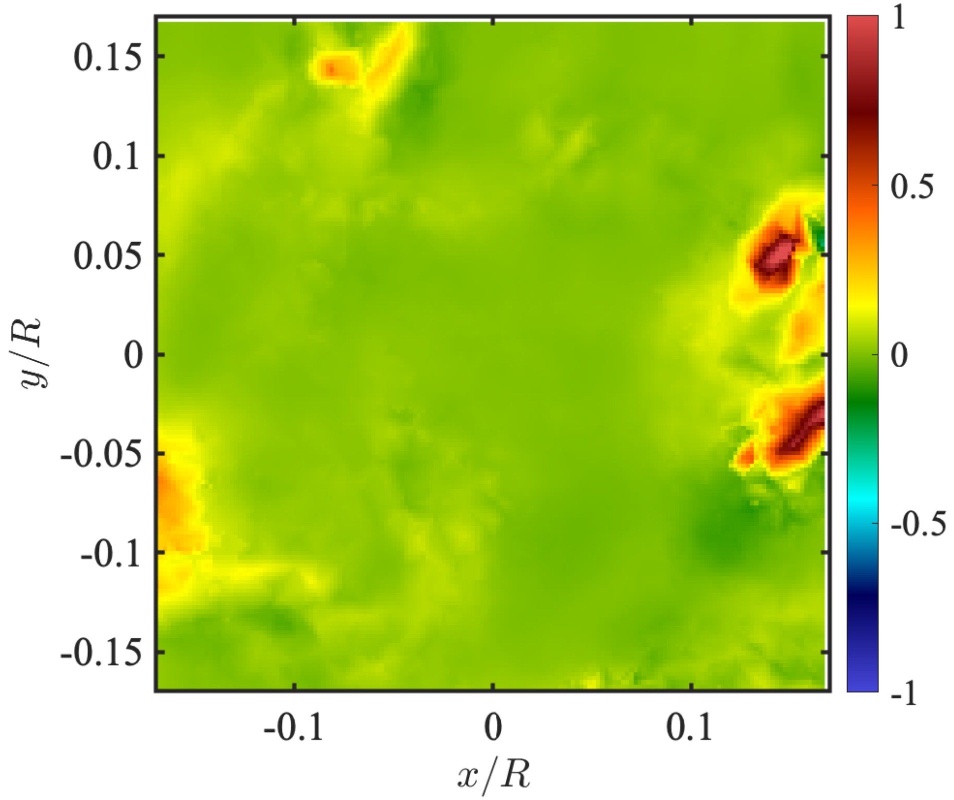
$$\delta_r X_{t_0,t}(x) = X_{t_0,t}(x+r) - X_{t_0,t}(x)$$

$$\Delta_\ell^{\pm\tau}(x,t) = \int d\boldsymbol{\xi}\ \phi^\ell(\xi)||\delta_\xi X_{t,t\pm\tau}(x) - \delta_\xi X_{t,t}(x)||^2$$

Backward vs forward pair dispersion

Not symmetric by time reversal

Lagrangian irreversibility



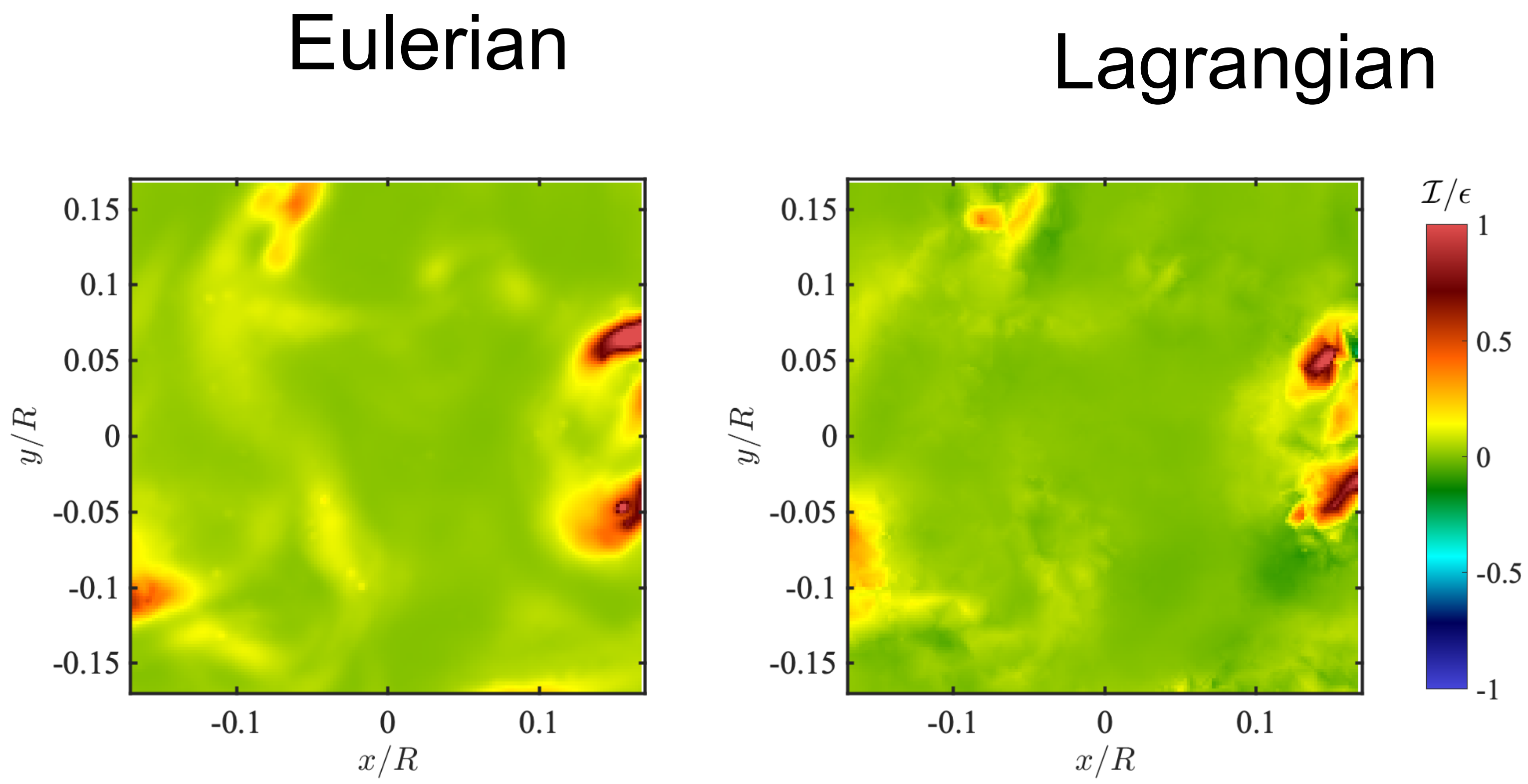
Irreversibility

$$D_L = \lim_{r,\tau \to (0,0)} (\langle (\delta X_r^{t+\tau})^2 \rangle - \langle (\delta X_r^{t-\tau})^2 \rangle)/6\tau^3$$

J. Jucha et al. PRL (2014)

Cheminet et al, PRL 2022

Eulerian vs Lagrangian local dissipation



Cheminet et al, PRL, 2022

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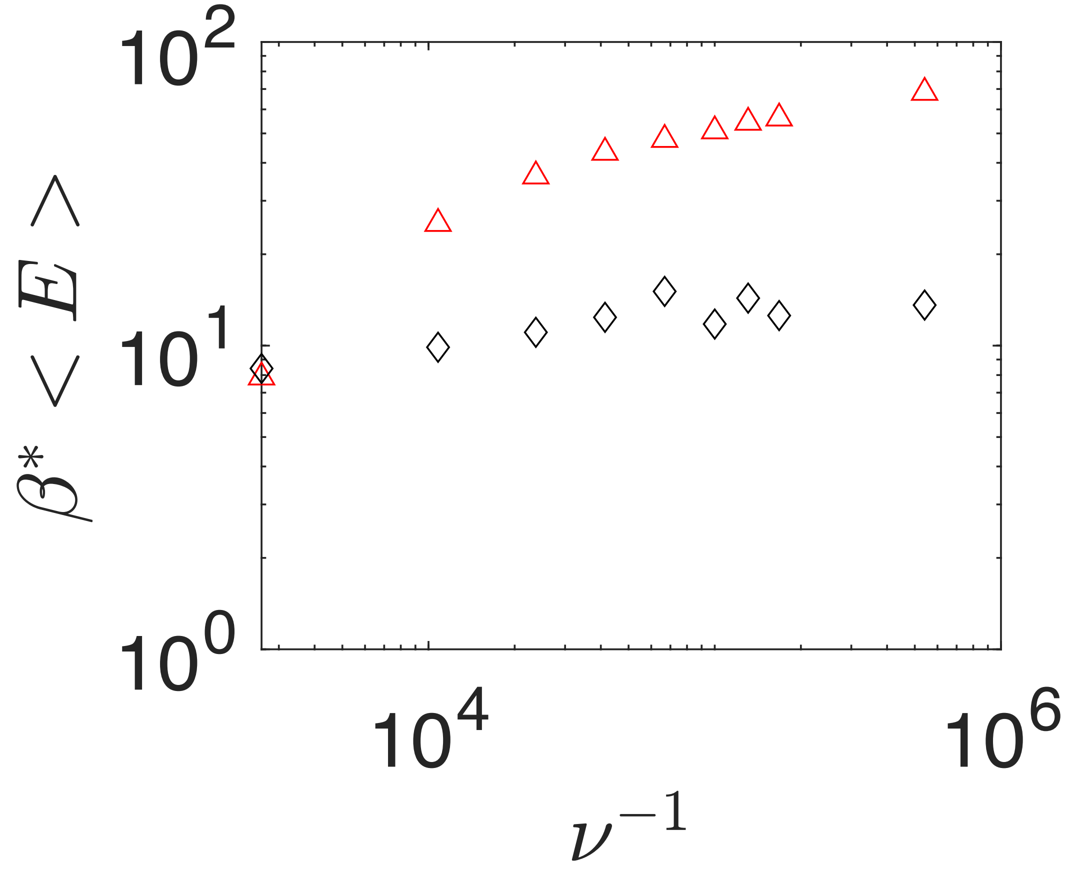
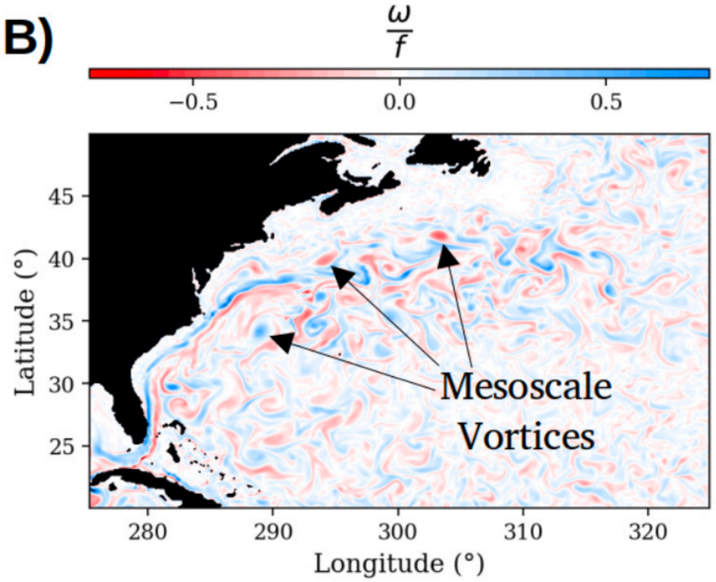
Dissipation of turbulence

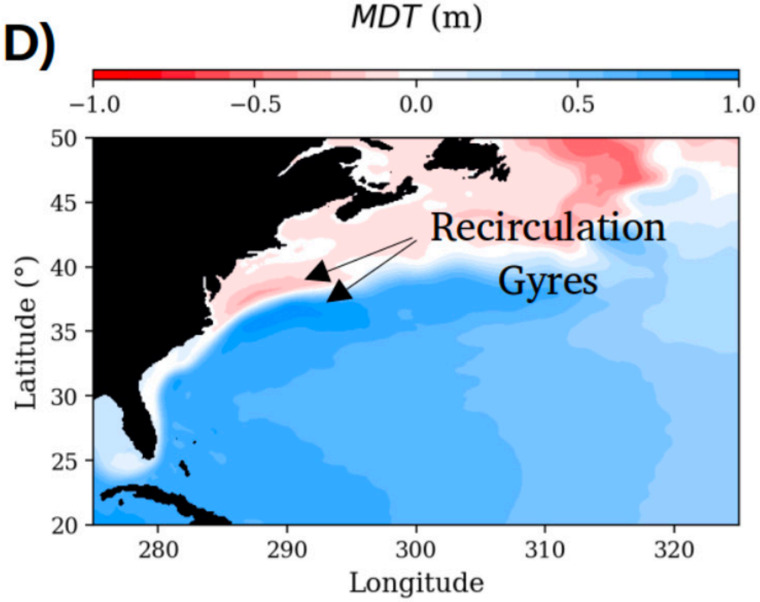
Coherent structures and Dissipation

Random scales contain most of the energy





Random

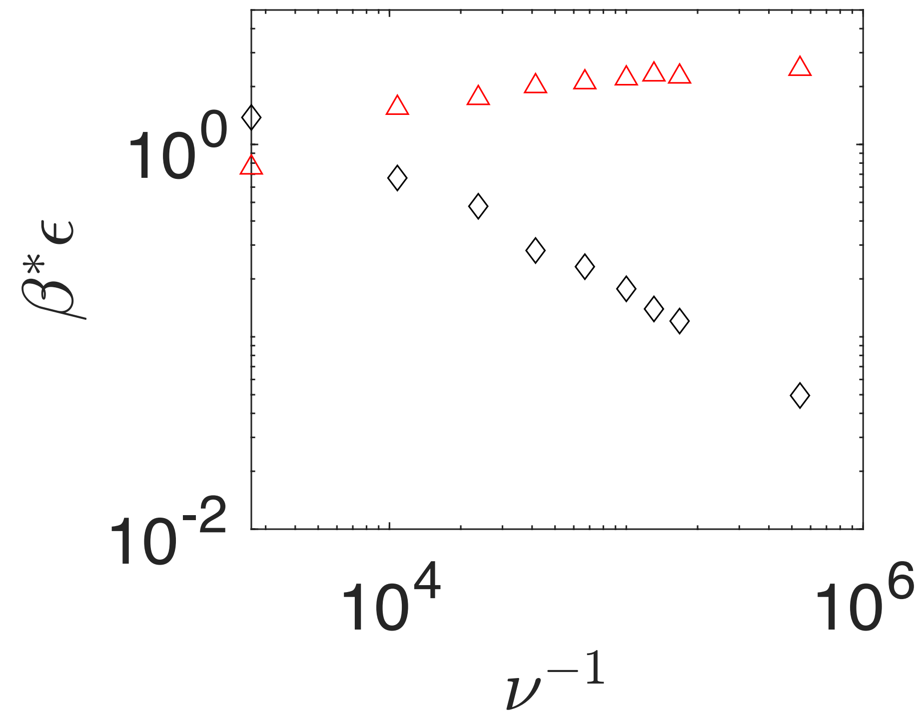


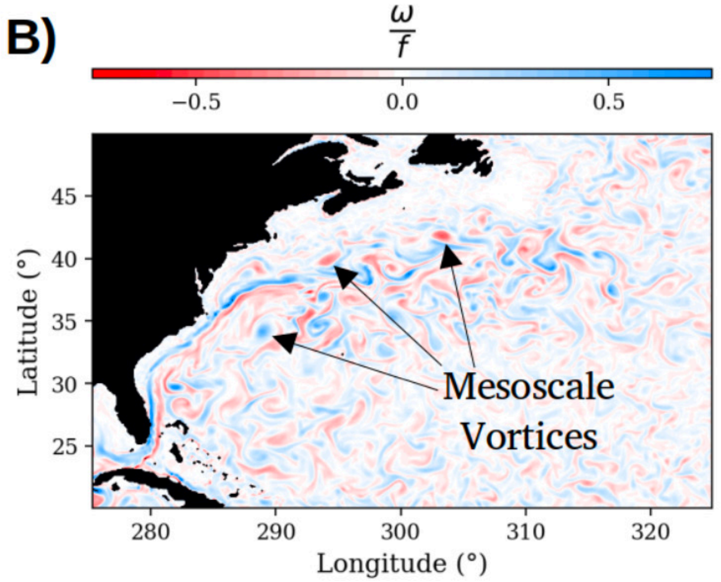
Mean

Miller PhD Thesis

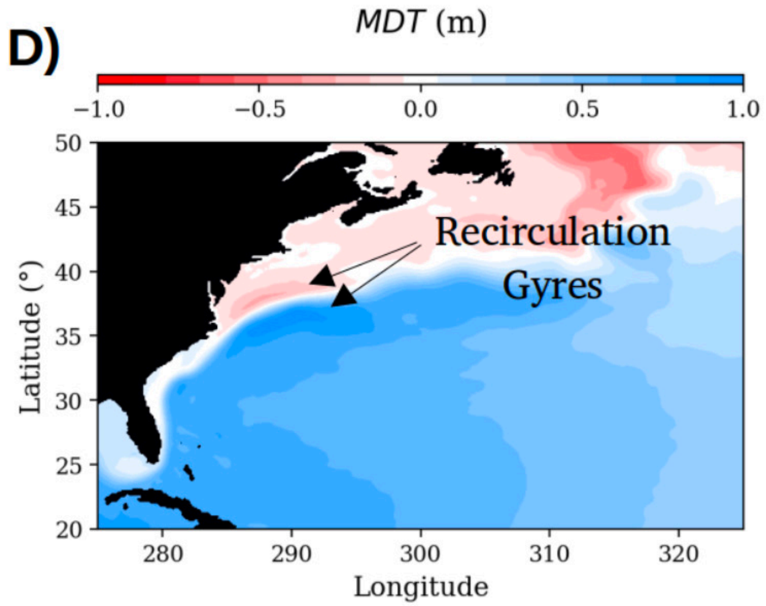
Coherent structures and Dissipation

Dissipation is due to random scales





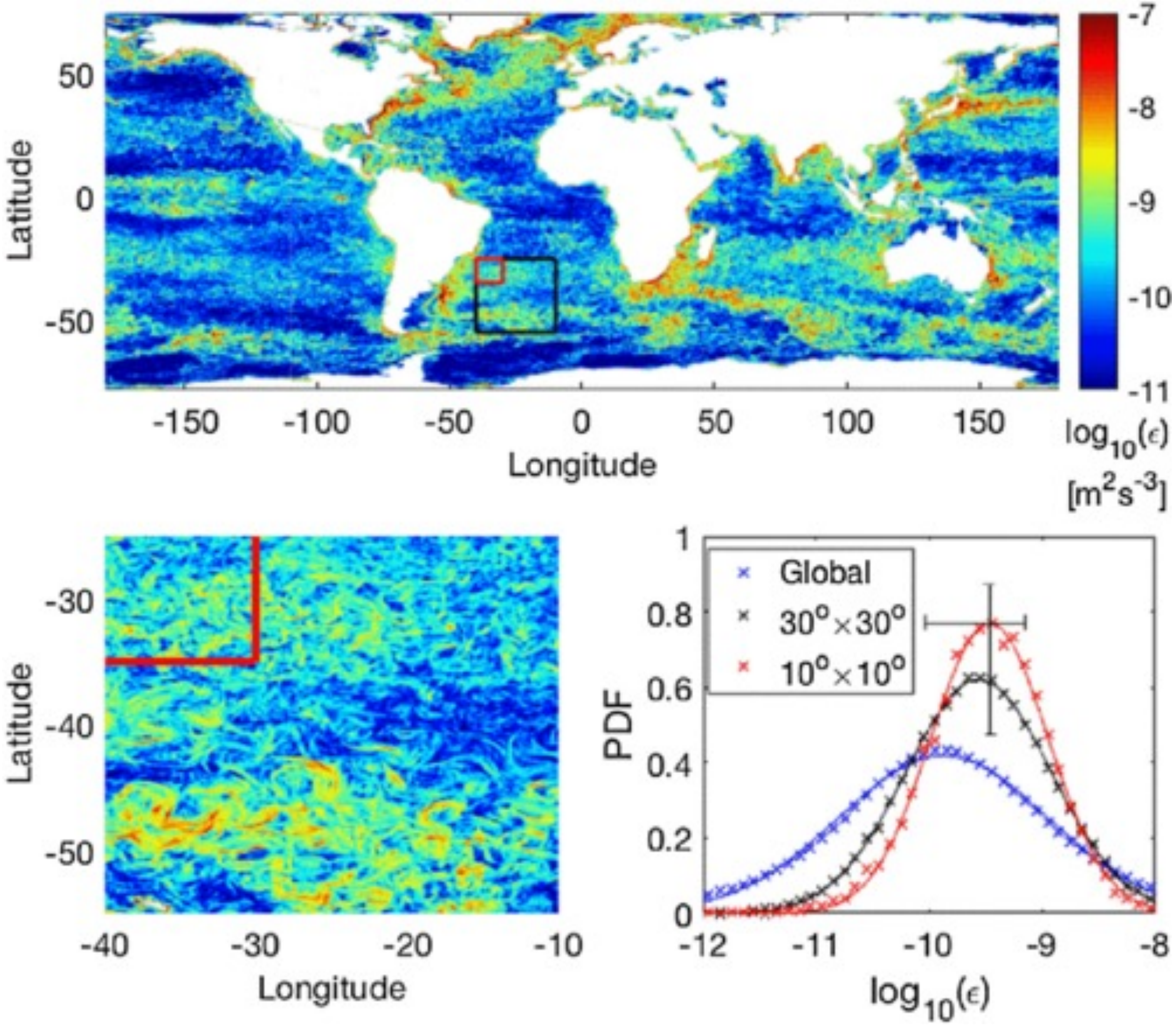
Random



Mean

Miller PhD Thesis

Statistics of Dissipation in the ocean



Dissipation is Log-normal

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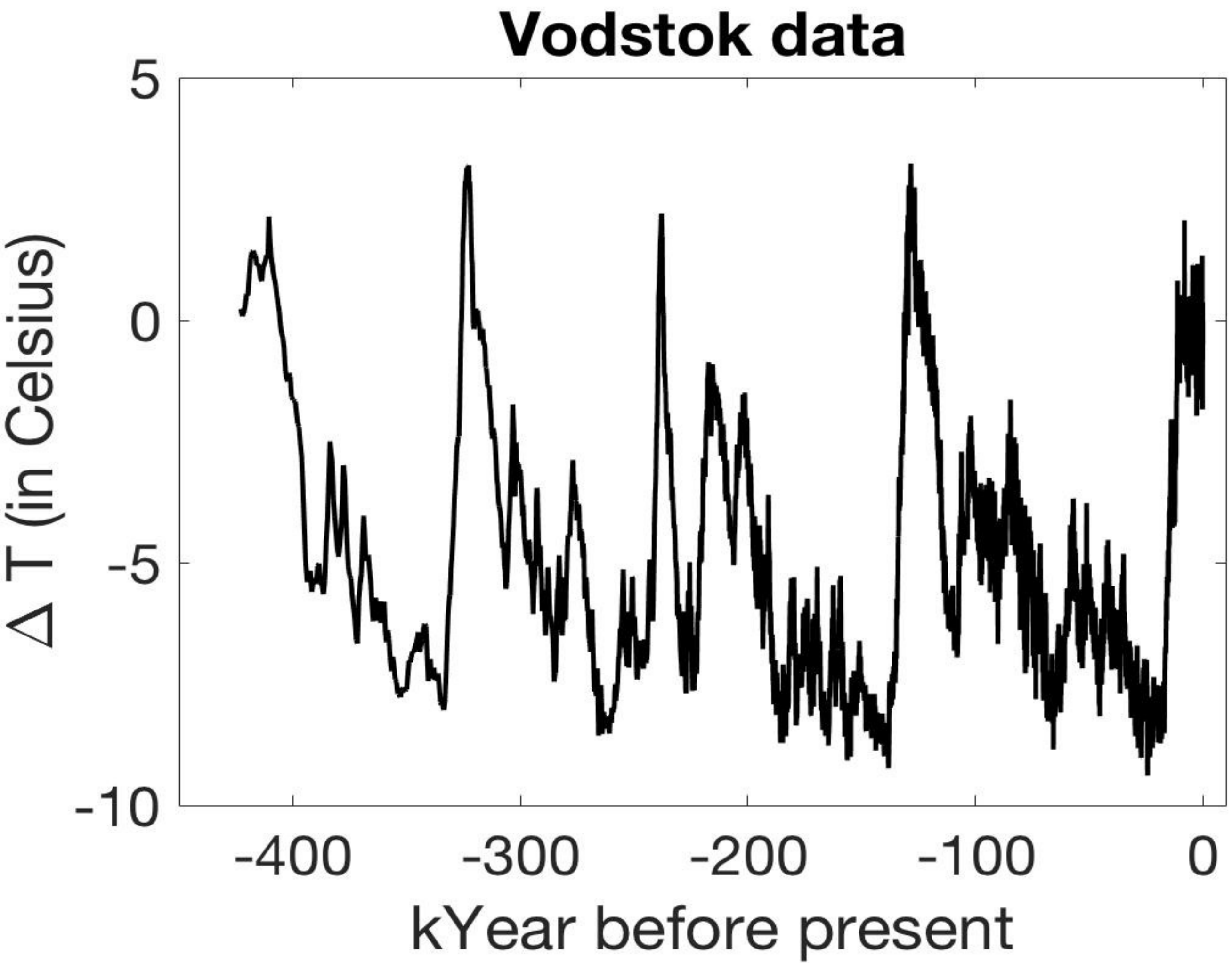
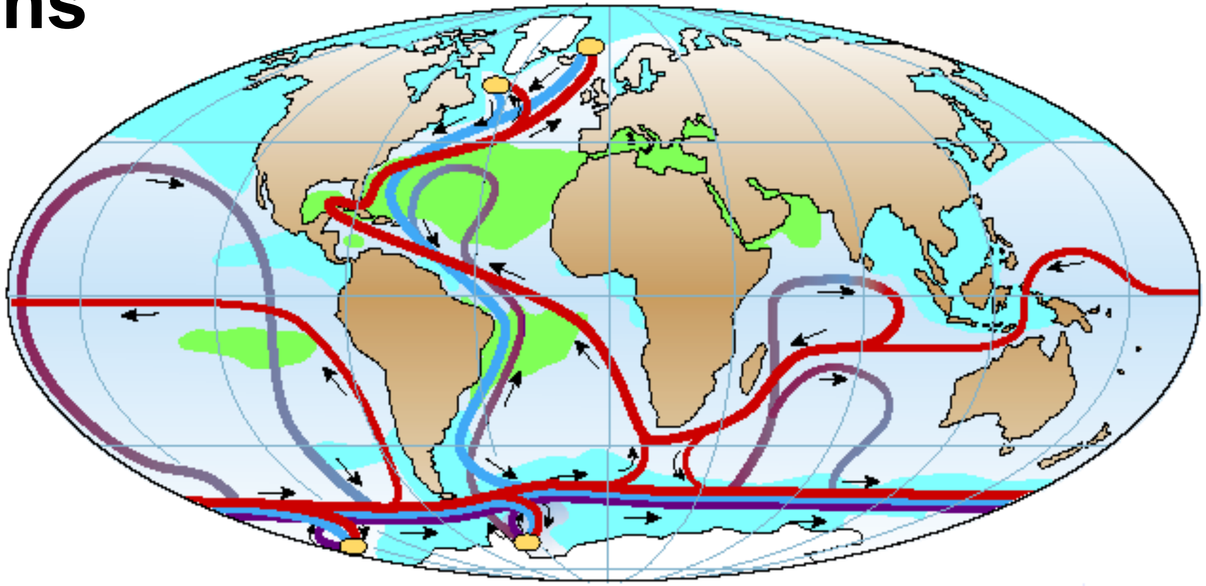
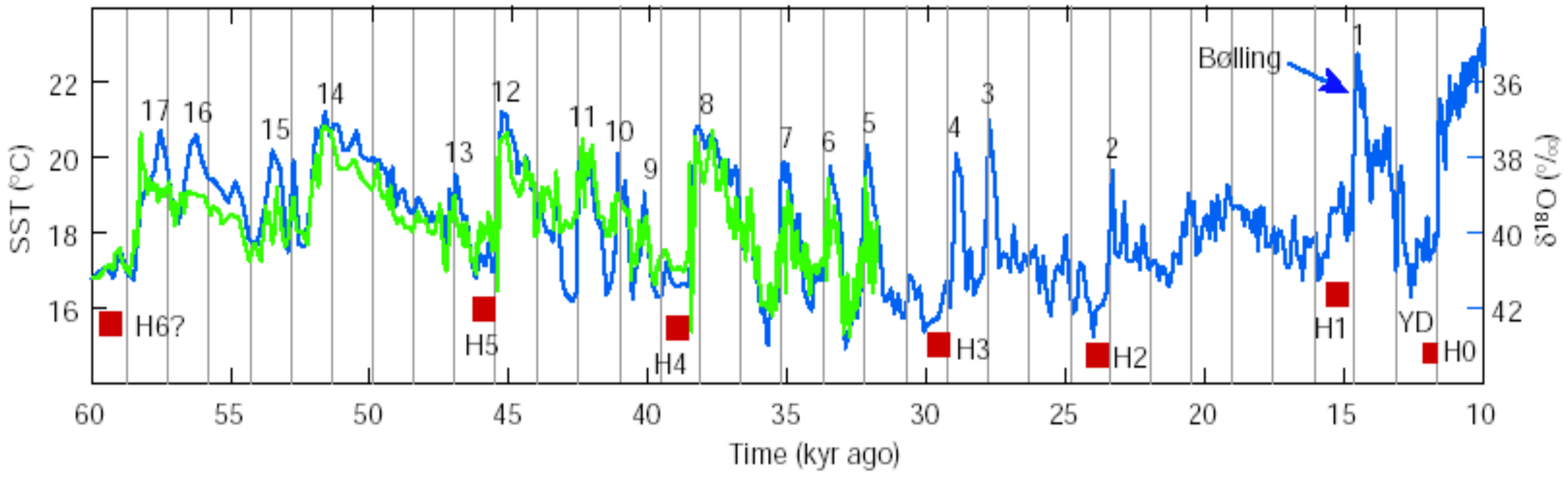
Bifurcations in turbulence

Global (Climatic) biburcations



Local (AMOC) bifurcations





years illustrate the tendency of D/O events to occur with this spacing, or multiples thereof.

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